

Do not write your name on the assignment. Write your name only on the back of this sheet of paper and staple your answers on the front of this sheet of paper. Your assignment will be typed, except graphs can be drawn by hand and mathematical equations can be done by hand. Failure to follow these directions will cost you 1 point on the assignment and failure to type it will cost you 10 points.

**For all questions, show all work.**

1) (40 points) Suppose labor costs  $\$2/L$  and capital costs  $\$6/K$ . If the production function is given by  $Q = 10K^{1/2}L^{1/2}$ , then treating  $Q$  as a constant, derive the total cost function of producing the quantity  $Q$ . Use that equation to derive the marginal cost and average total cost functions.

2) Assume that the demand curve for a duopoly is given by  $P = 63 - 2(Q_1 + Q_2)$  and the total cost function for firm  $i$  is given by  $TC_i = 13 + 3Q_i + 3Q_i^2$ .

A) (5 points) What is the profit function for firm #1?

B) (15 points) Using the Cournot-Nash assumptions, derive the profit maximizing best-response-function for firm #1 as a function of  $Q_2$ .

C) (15 points) What are the equilibrium outputs for the two firms, the industry price, and the profits for each firm in the Cournot-Nash model?

D) (5 points) Using your answer to part B, setup the Lagrangian for profit maximization for firm #1 assuming that firm #1 is the Von Stackleberg leader.

E) (20 points) Find the equilibrium outputs for the two firms, the industry price, and the profits for each firm in the Von Stackleberg model?