Write your name on the cover of the test booklet and nowhere else. Failure to follow these directions will cost you 1 point. The test has 150 points (to be scaled up to 200 points) and is scheduled to take 75 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I will allow some extra time, but I will not allow much.

Show all work on all questions.

1) (8 points) Do EITHER Part A OR Part B.
A) Suppose we wanted to minimize your income (so you can work as little as possible) subject to your utility is = 50. Your utility function is $U(D, F) = D^{1/3}F^{1/2}$, the price of a dog is $20/dog, and the price of a fish is $7/fish. Set up the Lagrangian and explain how you got it. **Do not solve it.**
B) What is the interpretation of λ in the utility maximization? Briefly explain your logic.

2) (10 points) Do EITHER Part A OR Part B.
A) Prove that profit maximization will yield MR = MC using calculus.
B) Is the $U(X, Y) = 12X^{1/3}Y^{1/3}$ a legitimate utility function? Prove your answer is true using the formal proof.

3) (24 points) Find all Nash equilibria in the following matrix. Prove that you found all and prove they are Nash equilibria. Does either firm have a dominant strategy? Find the cooperative equilibrium. Explain how you found it. What are the two players' secure strategies? How did you find them?

<table>
<tr><td rowspan="2">Payoff Matrix</td><td colspan="3" align="center">Patriots</td></tr>
<tr><td align="center">High price</td><td align="center">Medium Price</td><td align="center">Low Price</td></tr>
<tr><td rowspan="2">Steelers — High Price</td><td align="right">22

16</td><td align="right">23

14</td><td align="right">34

33</td></tr>
<tr><td align="right">27

18</td><td align="right">28

21</td><td align="right">20

30</td></tr>
<tr><td>Steelers — Low Price</td><td></td><td></td><td></td></tr>
</table>

4) (30 points) Suppose your utility function is $U(R, C) = R^{1/4}C^{1/3}$. The price of a racquet is $15 each and the price of chalk is $5/box. Find the utility maximizing consumption of racquets and chalk if you're your income is $350. Show all work. Do NOT worry about λ.

5) (36 points) Suppose that labor costs $6/hour and capital costs $4/unit and the firm's production function is given by $Q = K^{1/3}L^{1/2}$. Derive the total cost function which minimizes costs for a given output Q . Do NOT worry about λ. What are the marginal cost and average total cost functions? Show all work.

6) (42 points) Answer EITHER Part A OR Part B.
A) Suppose that both firms are facing the following demand and total cost functions:
$P = 121 - 2(Q_1 + Q_2)$ and $TC_i = 16 + Q_i + 3Q_i^2$. Use this to derive the best response function for Firm 1 and the equilibrium output for each firm, assuming that the firms are Cournot style firms. What are the outputs of the two firms and the market price? **Set up** the Von Stackelberg problem. Start solving it, but **stop after** you take all of the derivatives. Show all work.
B) Suppose your utility function is given by $U(D, M, H) = D^{1/4}M^{1/2}H^{1/4}$. The price of a dinner (D) $20/meal, price of a movie (M) is $10/ticket, and a hat (H) costs $30/hat. Your income is $990. You also have a time constraint of a dinner takes 3 hours while a movie takes 1.5 hours. You have 99 hours of time to spend. Find the utility maximizing levels of D, M, and H. Show all work. Do not worry about finding λ.