This review sheet is intended to cover everything that could be on the exam. However, it is possible that I may have inadvertently overlooked something. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones on the homework assignments and quizzes. There will **not** be a choice of questions to choose from. <u>If you do not show work, you will not get credit.</u>

The optional review session for this test will be Tuesday, 10/2 at 6:00 in Richardson 110.

Section 1.1: Be able to plot lines by finding points on the graphs. That is done by setting X equal to some value and finding the value of Y. Make sure X is on the horizontal axis and Y is on the vertical axis and there is a scale. To find the **y-intercept** which is where the graph crosses the y-axis, set X=0 and find out how much Y is. To find the **x-intercept** which is where the graph crosses the x-axis, set Y=0 and find out how much X is. **X-axis symmetry means that if (x,y) is on the graph then (x,-y) is also on the graph**. Test this by substituting -Y for Y in the equation. It is best to always put -Y in parentheses because some of the time you must do that. For example, if you had $Y^2=4+X$, then you would get $(-Y)^2 = 4+X$. This simplifies to $Y^2 = 4+X$. Therefore, there is x-axis symmetry. However, Y = 4+X would become -Y = 4+X. That is not the same equation, so there is no x-axis symmetry. **Y-axis symmetry means that if (x,y) I on the graph then (-x,y) is also on the graph.** Do this in the same method as x-axis symmetry except that you replace x with -x. **Origin symmetry means that if (x,y) is on the graph then (-x,-y) is also on the graph.** The test for symmetry about the origin, replace both X and Y with -X and -Y respectively.

Section 1.2: When solving equations for a variable like *X*, you must **do the same thing to both sides of the equal sign.** To move something to the opposite side of the equal sign, do the opposite operation of the current operation. For example, suppose you had $\frac{4X}{5} + 3 - X = 9$. If you wanted to move the 3 over, subtract (opposite of addition) 3. If you wanted to move the *X* over, add (opposite of subtract) *X* from both sides. If you wanted to move the 5, multiply (opposite of divide) both sides by 5. **Remember that you have to multiply every term on both sides.** So you would multiply 5 by the (4/5)*X*, the 3, the *X*, and the 9. Similarly, if you wanted to move the 4 over, you would divide (opposite of multiply) **all** terms by 4. Because multiplication and division are done to all terms, it is generally better to do the additions and subtractions before the multiplications and divisions, unless there is a common factor in all terms. If two things are equal, you can substitute one for the other.

Section 1.3: Learn the key words and their mathematical equivalence on page 97. For solving word problems. Follow the instructions below:

1) Read the problem – several times if necessary – until you understand what is being given and what is being asked for.
2) Let one of the unknown quantities be represented by a variable, typically the first letter of the unknown quantity. (Airspeed = A, etc.) You can use multiple variables. It may be helpful to think ahead to step 2B before deciding which unknown quantities to represent with variables.
    2A) Draw a figure when appropriate.
    2B) Look for formulas that connect the unknowns.
3) Form equations that relate unknown quantities to known quantities. Make sure you have the same number of equations as unknown variables.
4) Solve the equations for all unknown quantities. Make sure you write the units for the variables.

5) Make sure you check your answers with the word problem initially given and in the equations in steps 2B and equation 3.

For distance and/or rate problems, it is helpful to remember Rate = Quantity/ Time, i.e. R = D/T. (I used D for quantity because the quantity is normally a distance.) Similarly, D = R*T, and T = R/D. If you only remember one, you can derive the other two by division or multiplication.

For mixture and/or percentage problems, it is helpful to remember $\frac{\%_1 * Q_1 + \%_2 Q_2}{Q_1 + Q_2} = \%_{desired}$. The "1" subscript is for the first item mixed and the "2" is for the second item mixed. Q is the quantity of each.

For interest problems, remember interest = Prt where P = principle, r is the interest rate, and t = time.

Profit is calculated as revenue-costs where revenue is price*quantity.

Section 1.4: If you can get the equation as $aX^2 + bX + c = 0$, then **factor the left-hand side**. Set each factor = 0. That gives you two solutions. If you have $X^2 = a^2$, then you can **take the square root of both sides remembering to add $\pm$** and get $X = \pm a$. Completing the square is usually not worth your time. If nothing works quickly, you can always use the **quadratic formula**. Memorize it. It is very helpful. Always test your answers in the equation. Notice that some times one answer is not usable. For example, if we are talking about room sizes, one solution is likely to be that one of the variables is negative. If your numbers do not work in the original equation, you have done the mathematics wrong unless you are looking at absolute values. Radicals are the same as the fractional exponents. It is often handy to convert them into fractional exponents before working with them. Note that if <u>there is only multiplication inside the radical</u>, you can switch the order of the exponent and radical like $\sqrt[5]{X^3} = (\sqrt[5]{X})^3$ because both are $X^{3/5}$. **Never leave a larger exponent inside the radical than the index** (the 5 in the example above is the index). So $\sqrt{X^5}$ can be split into $\sqrt{X^4} * \sqrt{X} = X^2\sqrt{X}$. **Make sure there are no radicals in the denominator of a fraction.** To get rid of it, multiply the top and bottom of the fraction by the same thing that will cause the denominator to not have a radical. For example, $\frac{3}{\sqrt{2X}}$, can be simplified by multiplying the top and bottom by $\sqrt{2X}$. This would give $\frac{3\sqrt{2X}}{2X}$. **Do not leave a fraction in a radical**. Split it into two radicals and get rid of the radical in the denominator. For example: $\sqrt{\frac{3}{X}} = \frac{\sqrt{3}}{\sqrt{X}} * \frac{\sqrt{X}}{\sqrt{X}} = \frac{\sqrt{3X}}{X}$. **You will not** be held responsible for n-th degree radicals in the denominator or the addition of radicals in the denominator. Examples of these are $\frac{1}{\sqrt[3]{X}}$ and $\frac{1}{\sqrt{X} + \sqrt{3}}$.

Section 1.5: Imaginary numbers look scary but are actually not hard. They are called imaginary because i= the square root of -1. Complex numbers are a+bi where a and b are real numbers. **Treat i as if it were any other letter like x.** Only add the numbers in front of it if the power of i is the same. In other words, 3i + 2i = 5i, but you cannot add 3 + 2i and simplify it. It is simplified. The only difference between i and x is that $i^2 = -1$. This also makes it easier to simplify fractions. For example, if you have $\frac{3 + 2i}{1 + 7i}$ then multiply by the conjugate over itself. In this case, that is (1-7i)/(1-7i). **Remember to FOIL the numerator and the denominator.** The denominator becomes $1 - 7i + 7i - 49i^2 = 1 + 49 = 50$. Always switch the sign on the operation in the middle and the i terms will cancel each other out. Of course the i^2 is -1, so that becomes a number. Also, remember that $(-a)^{1/2} = a^{1/2}(-1)^{1/2} = a^{1/2}i$.

Section 1.6: **For bigger polynomials, factor.** First factor out common factors. Convert to quadratic form equations. For example $X^4 + 4X^2 + 4 = 0$ can be converted into an easier form by letting $U = X^2$ and substituting it in. That gives $U^2 + 4U + 4 = 0$. Solve for U. Then solve for X. Note that if the polynomial is of degree n, you should get n solutions. So, this example should have four solutions. In this case some are duplicates. It factors as $(U+2)(U+2)=0$. So U = -2 or U = -2. That means $X = \pm(-2)^{1/2} = \sqrt{2}i, or -\sqrt{2}i$. That looks like only two solutions, but technically it is four because there are those two for both of the solutions for U=-2. **When solving absolute values** like $|X+2|=5-X$ write it as the positive value = the other side or the negative value = the other side like $X+2=5-X$ or $-(X+2)=5-X$. **Note to distribute the negative side through to get -X-2.**

Section 1.7 Inequalities are easy to work with, but require a tiny bit more work than equalities. When graphing, if there is a $<$ or $>$ sign, then you use the (or), and if there is a $\leq$ or $\geq$, then use [or]. Therefore, (2,4] is $2<X\leq 4$. When plotting unions, (e.g., $X<5$ or $X>7$) plot everything that is in either one or in both. (Just like with the union of marriage, everything now includes all that both had before.) When plotting intersections, (e.g., $X>5$ and $X<7$) you only plot what is in both. (Just like a street intersection, it has to be in both sets.) When solving inequalities for a variable, you must do the same procedures as for equality. However, there is one major difference. **If you either multiply or divide by a negative number, reverse the inequality**. If you start with $-3<-X<5$, and you multiply by -1, you would get $3>X>-5$. (Note that if you forgot to reverse the sign, you would get $3<-5$.) As I have just illustrated in this example, if you have a double inequality, you must do the same procedure to **all** three sides of the inequality. (However, you can break it into two inequalities, solve them, and reassemble them. That method will always work, but usually requires two or more extra steps.). For example you will have to know how to plot $|X-4| <6$ or $|X+2|\geq 5$ or $|X-3| < 3X$. Absolute values convert negative numbers to positive only and leave positive numbers alone. **Note that $|4-2| \neq |4|+|-2|$ and $|-4+2|\neq|-4|+|2|$. You cannot distribute the absolute values through.** When plotting $|X-4|<5$, you start at 4 and go five units in both directions. You find all points between them because of the $<$. If you are plotting $|X+4|>5$, then start at -4 and go five units in both directions. Fill in all points **except** those between the two end points. When solving $|f(X)| < g(X)$ then you have two equations $f(X)<g(X)$ and $-f(X)<g(X)$ which could be written as $f(X) >-g(X)$. If you have $\leq$, then replace $<$ with $\leq$ and $>$ with $\geq$. When solving $|f(X)| > g(X)$ then you have two equations $f(X)>g(X)$ or $-f(X)>g(X)$ which could be written as $f(X) <-g(X)$. If you have $\geq$, then replace $<$ with $\leq$ and $>$ with $\geq$. **Test all of your "solutions" because usually the absolute value will give you more values than are usable. That is because you use the equation for if f(X) is positive when the end result is negative or vice versa.**

Section 1.8: **Factor the polynomial as you did in Section 1.6 and set equal to zero**. **That tells you the endpoints of the regions.** For example, $X^2 - X - 6 >0$. That gives $(X-3)(X+2) >0$. Setting it equal to zero says the border points are X = 3 and X = -2. So X is in none, one or more of the following areas $(-\infty,-2)$, $(-2,3)$, $(3, \infty)$. Note that if it had been $\geq$ then the values could include -2 and 3 so you would have [or] in the corresponding places. Check all three areas to find which work. Try -3 for the first area, 0 for the second area and 4 for the third. You get $9+3-6=6>0$, $0-0-6= -6>0$, and $16-4-6 = 6>0$. So the first and third areas work. Thus, the solution is $(-\infty,-2)$ or $(3, \infty)$. If the inequality had been reversed, then it would be $(-2, 3)$. There are some cases all numbers work. For example, $X^2+4X+4\geq 0$. You get $(X+2)(X+2)\geq 0$. So there are two intervals and the border point $(-\infty,-2)$, $(-2, \infty)$, and -2. Testing all three work. In some cases no answers work. For example $X^2+4X+4<0$. If you have a fraction with X in the denominator work as you would otherwise. However, make sure you know what values X cannot have and check the numerator's roots. For example, $(X-8)/(X+5)\leq 0$, we know X cannot be -5 and the root for the numerator is X=8. So, check a number in $(-\infty,-5)$, $(-5,8]$, and $[8, \infty)$. The () is next to -5 because it cannot work and the [] are next to 8 because of the = in $\leq$. Check -6, 0, 8, and 9 because you need to check a number in each range and the border which is possible. -6 does not work, 0 does, 8 does, and 9 does not. So the answer is $(-5,8]$.