Wilfrid W. Csaplar Jr. College Algebra Review Sheet for Exam 3

This review sheet is intended to cover everything that could be on the exam. However, it is possible that I may have inadvertently overlooked something. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones on the homework assignments and quizzes. There will **not** be a choice of questions to choose from. <u>You will not get credit for answers given without work.</u>

The optional review session for this test will be Tuesday, 10/30 at 7:00 hopefully in the normal room.

Note that much of this material was in Test #4 the last time I taught this because of the book used.

Section 2.1: The **slope-intercept form** of a line is y = mx + b where m is the slope and b is the y-intercept. *The closer |m| is to zero, the flatter it is. |m| = 1 is diagonal. If m<0 it slopes down, and m>0 slopes up. Plot a slope of m = a/b by going up a units and right b units.* The **slope is found from two points** using Δy/Δx, i.e., rise/run. The **point-slope form** is $y-y_1 = m(x-x_1)$ where (x_1, y_1) is a point on the line. To get from one form to the other, just use algebra. **Parallel lines** have the same slope. **Perpendicular lines** slopes are m and -1/m. *The summary on page 178 is helpful.*

Section 2.2: **Functions** go from elements of the **domain** to elements of the **range**. Each element of the domain (**independent variable**) is mapped to exactly one element in the range (f(x) or the **dependent variable** y), but two different elements of the domain can go to the same element in the range. Function notation is f(x) = ... *Note that sometimes other letters are used for functions, most commonly, g(x) and h(x).* **A piecewise function** is one where there are different functions depending upon where in the domain you are. For example, $f(x) = \begin{cases} x^2 & x \le 3 \\ x & x > 3 \end{cases}$

Make sure you are using the correct part of the function for the value of x you are using. To find the **zeros of a function**, you set f(x) = 0 and solve for x. Similarly to find where two functions cross (i.e., are equal) set f(x) = g(x) and solve for x. The **domain of the function** can be found by finding all points where the function is defined. *For example, if f(x) = 1/x, then x cannot = 0.*

Section 2.3: The graph of a function is done the same way you would graph y=... To test to see if a graph is a function, then do the **vertical line test**. If <u>any</u> vertical line crosses the graph more than once, then it is not a function. **Increasing functions, decreasing functions,** and **constant functions** are just what they sound like. **Relative (local) minima** and **relative (local) maxima** are where the line is flat and increasing on one side and decreasing on the other side.

Section 2.4: **Plotting linear functions** is just like what we described in Section 2.1. Know what the **constant function** and the **identity function** are. The **squaring function** is $f(x) = x^2$. *It us u-shaped and has its minimum at the origin*. Know what the graphs of $\mathbf{f(x)=x^3}$, $\mathbf{f(x)=x^{1/2}}$, and **f(x) = 1/x** look like. They are called the **cubic, square root,** and **reciprocal functions** respectively. Know what the **greatest integer function** (f(x) = ⟦x⟧) does and how to plot it. *Hint, for negative numbers* ⟦x⟧ *rounds down. So* ⟦-2.4⟧ *becomes -3.*

Section 2.5: When it comes to **shifting graphs**, adding a constant c to the function will move it up c units. Therefore, g(x) = f(x)+c will move up c units from the original f(x). However, replacing x with x+c will move it left c units. Therefore, g(x) = f(x+c) will move left c units from the original f(x). If you do g(x) = -f(x), you **reflect it over the x-axis**. If you do g(x) = f(-x), then you **reflect it across the y-axis.** *If you multiply f(x) by a constant then one of four things will happen. If the constant is >1, then it gets stretched up. If the constant is between 0 and 1, then it is flattened down. If the constant is between -1 and 0, then it is flattened down and upside down. If the constant is less than -1, then it is stretched downward and flipped over.*

Section 2.6: Know that **(f+g)(x)**=f(x)+g(x), **(f-g)(x)**=f(x)-g(x), **(fg)(x)** = f(x)*g(x), **(f/g)(x)**=f(x)/g(x) and the **composite function** is **(f∘g)(x)** = f(g(x)).

Section 2.7: An **inverse function f^{-1}(x),** goes from the range of f(x) to the domain in such a way that f^{-1}(f(x)) = x. and f(f^{-1}(x)) = x. You can verify the inverse by doing either of those calculations. A function has an inverse if and only if it is a **one-to-one function**. That means every element in the range of f(x) has exactly one element of the range which maps into it. You can check to see if this is true using the **horizontal line test**. *Note the graph of the inverse function is a reflection the original function about the identity function, f(x) = x.* To find the inverse of a function, you can do it the way the book says, or do the following: *1) Replace x with y. 2) Replace f(x) with x. 3) Solve for y. 4) Replace y with $f^{-1}(x)$. 5) Verify your answer is correct.* Note that you can combine steps 1) and 2) into one step. Be careful about the domain and range for f and f^{-1}.

Section 3.1: Know what a **polynomial of degree n** is and how to recognize one. Know the special case of a **quadratic function, a.k.a., binomial or polynomial of degree 2.** The **standard form** of a quadratic function is f(x) = a$(x-h)^2$ + k where a ≠ 0, the **axis of symmetry** is x = h, and the height of the **vertex** is k. *The a determines if it is a right-side up parabola (a>0) or upside-down (a<0) and whether it is stretched vertically (|a|>1) or shrunk vertically (|a|<1).* The **minimum or maximum value** is found at the vertex. The sign of a tells you whether it is a maximum or a minimum. *To get the function in standard form you do the following. Start with $f(x) = ax^2 + bx + c$. Factor a out of the first two terms. This yields $a(x^2 + (b/a)x) + c$. If we started with $f(x) = 2x^2 + 6x + 4$, we get $f(x) = 2(x^2 + 3x) + 4$. Then find the term to add and subtract inside. You do that by dividing the number in front of x by 2 and squaring it. In this case that is $(3/2)^2 = 9/4$ which in this case gives $f(x) = 2(x^2 + 3x + 9/4 - 9/4) + 4$. Bring the negative number out of the parentheses remembering to multiply by the external coefficient first. That gives $f(x) = 2(x^2 + 3x + 9/4) - 9/2 + 4$. Note that if you did it correctly, it factors to $(x+3/2)^2$ where the 3/2 is the number in front of the x divided by 2. So, $f(x) = 2(x+3/2)^2 - 1/2$.*

Section 3.2: Polynomials do not have discontinuities nor sharp points. Note the general form of a polynomial of degree n will either start rising and end rising or start falling and end falling if n is odd. Note the general form of a polynomial of degree n will either start rising and end falling or start falling and end rising if n is even. The **leading coefficient** tells you which pattern of those is the correct one. An n-degree polynomial has **at most n real zeros** (Section 3.4 says there are exactly n complex zeros.). There are at most n-1 **turning points**. Note that the book has a nice table on Page 274. The **intermediate value theorem** is easier than it would first appear. Between two points x = a & b, there must be a points whose value is any value between f(a) and f(b).

Section 3.3: **Long division**. Just look at the first terms of both polynomials. Do it just like long division of numbers. For **synthetic division**, if you are dividing by (x-k) put k outside. Then put the coefficients inside remembering to put a zero if there is no term. So $x^2 + 5$ would be 1 0 5 inside. Then bring the first number down. Multiply it by k and place that up on the next line. Then add and repeat. So $(x^2+5)/(x+2)$ is done as:

```
– 2| 1  0  5
   |___-2  4_
     1 -2  9
```

So the answer is x-2 +9/(x+2) or x-2 R 9. Note that doing synthetic division will give you R = f(k). Notice that since we divided by (x+2) we used -2 outside. **R = f(k)** is the **Remainder Theorem.** That should help you to find the zeros.

Section 3.4: A polynomial of degree n can factor as $a_n(x-c_1)(x-c_2)...(x-c_n)$ where a_n is the coefficient on x_n, and **c_i are the n complex roots a.k.a., zeros.** Therefore, there are n complex roots. If you are trying to find **rational roots (x = p/q)** then try all positive and negative numbers of the form p/q where p divides into the constant and q divides into a_n. Note, you can use synthetic division to test to see if the roots are viable, you do not need to plug them into the equation. For complex numbers the **complex conjugate** of a root is also a root. So, if 4 + 2i is a root, then 4-2i is too. To factor a polynomial, find all of the zeros (synthetic division will help for rational roots) then the polynomial factors as $a_n(x-c_1)(x-c_2)$... The **Descartes Rule of Signs** is helpful. It has two parts. The number of *positive* real zeros of f(x) is either the number of times the sign changes or less than it by an even number. The number of *negative* real zeros of f(x) is either the number of sign changes of f(-x) or less than that by an even number. For example, if $f(x) = x^5 + x^4 -x^3 -x^2 + x -1$ has three sign changes between x^4 and $-x^3$, between $-x^2$ and x, and between x and -1. So it will have either 3 or 1 positive real zeros. If we replace x with -x, then we get $f(-x) = -x^5 + x^4 + x^3 - x^2 - x -1$. (All odd exponent terms change sign but even terms do not.) Therefore, there are two sign changes, between $-x^5$ and x^4, and between x^3 and $- x^2$. Therefore, there are either 2 or 0 negative real zeros. Finding an **upper bound** of the zeros can be found using synthetic division. If $a_n > 0$ $c>0$, and doing synthetic division by c yields only numbers which are positive or zero, then c is an upper bound of the zeros. Finding a **lower bound** of the zeros can be found by using synthetic division. If $a_n>0$, $c<0$, and the synthetic division alternates signs, then c is a lower bound for the zeros. Note that is the case of the synthetic division on top of this page. Therefore, all real zeros of $f(x) = x^2 + 5$ must be greater than -2.

Section 3.5: A **least squares regression line** is one where you try to fit a straight line through a series of data points in such a way that when we add the squares of the error terms, we get as small a number as possible. A relationship between two variables is said to be **directly proportional** if y=kx. A variable y is said to **vary directly as the nth power of x if $y = kx^n$**. A variable y is said to **vary inversely or is inversely proportional** to x if y = k/x. A variable z is said to jointly vary with x and y if z = kxy.