Wilfrid W. Csaplar Jr. College Algebra Review Sheet for Exam 4

This review sheet is intended to cover everything that could be on the exam. However, it is possible that I may have inadvertently overlooked something. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones on the homework assignments and quizzes. There will **not** be a choice of questions to choose from. <u>You will not get credit for answers given without work.</u>

The optional review session for this test will be Thursday, 11/29 at 7:00 in the normal room.

Section 4.1: **Rational functions** are ones which can be written as **F(X) = N(X)/D(X)** where N(X) and D(X) are polynomials of degree n and m respectively. <u>Hints: the function names come from "N"umerator and "D"enominator and the degree n corresponds to N(X).</u> **Vertical asymptotes** are lines like X = a where F(a) is not defined because D(a) = 0, N(a) ≠0 (See Section 4.2 for more on N(a)≠0.), and as X gets closer to a, F(X) goes to either ∞ or -∞. A **horizontal asymptote** are lines like Y = b where F(X) approaches b when X goes to either ∞ or -∞. *You find the asymptotes in the following manner. <u>Make sure N(X) and D(X) have no common factors</u> by factoring both and canceling out common factors. <u>Note that you need to remember that F(X) is not defined at the points where the common factors =0.</u> The vertical asymptotes are the zeros (roots) of D(X). There are three possibilities for horizontal asymptotes. 1) If n < m, (degrees of N(X) and D(X)) then the asymptote is Y=0. 2) If n=m then there is an asymptote of Y =* a_n/b_n *where* a_n *and* b_m *are the first terms of N(X) and D(X) respectively. 3) If n>m then there is no horizontal asymptote.*

Section 4.2: I am going to slightly modify the list of steps for plotting rational functions to consider slant asymptotes which are later in the section. If F(X) = N(X)/D(X), then:

1) Simplify F(X) *by factoring both N(X) and D(X) and discarding common factors. <u>Remember those common factors so you can put a hole in the line there.</u> For example, if F(X) = 3(X-2)/(X-2) then it simplifies to F(X) = 3, but is not defined at X=2. So you draw a horizontal line at Y =3 but there is a hole at (2,3). <u>Leaving F(X) factored will help with steps 3 and 4.</u>*
2) Find and plot the Y-intercept (if it exists) by finding F(0). *It won't exist if D(0) = 0.*
3) Find the X-intercepts by finding the zeros (roots) of N(X) (if they exist) by solving N(X) = 0 for X. Plot the point(s).
4) Find the vertical asymptote(s) (if they exist) by solving the equation D(X) = 0 for X. Draw the vertical asymptote(s) *as vertical dotted line(s).*
5) Find the horizontal asymptote (if it exists) as described at the end of Section 4.1. Plot it *as a horizontal dotted line*.
6) Find the slant asymptote (if it exists) by dividing N(X)/D(X) using long division. Plot them *as dotted lines. If D(X) is of the form D(X) = X - a, then you can use synthetic division. <u>Only do this step if the degree of N(X) is one greater than the degree of D(X).</u> For example, if N(X) =* X^2*+3X +5 and D(X) = X+1, then the long division will give you X+2 +3/(X+1), a.k.a. X+2 R3. The slant asymptote is Y=X+2.*
7) Plot at least one point <u>between</u> and one point <u>beyond</u> every X-intercept and vertical asymptote. *Suppose the function has X-intercepts at X = -4 and X = 6 and a vertical asymptote at X= 3. You want to find F(X) at one point in each of these intervals (-∞, -4), (-4, 3), (3, 6), and (6, ∞). These points will help you to figure out what the graph is doing between the intercepts and asymptotes. I always try to find numbers which are integers close to zero because they work best. So, I would use -5, 0, 4, and 7 respectively.*
8) Draw a smooth line connecting the dots *making sure to not cross any asymptote. Does this remind*

you of pre-school?

Section 4.3: Know the **conic sections** but not the degenerate conic sections. **Parabolas** are all points equidistant from a point called the **focus** and a line called the **directex**. In the **standard form**, the **vertex** is at the origin and the equation is either $X^2 = 4pY$ or $Y^2 = 4pX$. The focus is at (0,p) or (p, 0) respectively. And the directex is at Y = -p or X = -p respectively. *The smaller |p| is, the sharper the curve and a negative p will turn it down or left for* $X^2 = 4pY$ *and* $Y^2 = 4pX$ *respectively.* An **ellipse** has two **foci**, a **center**, a **major axis**, a **minor axis**, and **vertices**. It is all points the same sum of the distances from the foci. The **standard form** for an ellipse is either $X^2/a^2 + Y^2/b^2 = 1$ for a horizontal major axis or $X^2/b^2 + Y^2/a^2 = 1$ for a vertical major axis. In both cases, a>b. The vertices on the major axis are where that variable =a and on the minor axis where the variable =b. The foci are at (-c, 0) and (c, 0) for a horizontal major axis and (0,-c) and (0, c) for the vertical major axis. In both cases, $c^2 = a^2-b^2$. **Hyperbolas** are of the form $X^2/a^2 - Y^2/b^2 = 1$ or $Y^2/a^2 - X^2/b^2= 1$. The vertices are (-a, 0) and (a, 0) in the former case and (0,-a) and (0,a) in the latter case. The foci are (-c,0) and (c, 0) for the former and (0,-c) and (0, c) for the latter. Here $c^2 = a^2 + b^2$ The asymptotes are Y=(-b/a)X and Y=(b/a)X in the former case and Y=(-a/b)X and Y=(a/b)X in the latter. <u>The formulas for the hyperbolas ad ellipses are easily confused. Note the hyperbolas have - and the ellipses have +. However, the sign in the c equation is the opposite of what it is in the general formula. In other words, the equation for c is - for ellipses and + for hyperbolas. For the asymptotes of the hyperbola, find the points (a, b), (-a, b), (-a, -b), and (a, -b) if a^2 is under X and the points (b, a), (-b, a), (-b, -a), and (b, -a) if a^2 is under Y. In other words, if a is under X put it in the X coordinate and if it is under Y, put it in the Y coordinate. Draw an X through the four points and crossing at the origin. Those are the asymptotes. The vertices are at "a" and "-a" on the appropriate axes.</u>

Section 4.4: Basically this is when you move the conic sections from Section 4.3. It can all be simplified to two statements. <u>1) If you just replace Y with Y-k, then move all points up k units</u>. *That includes the center, vertices, foci, directex,, intercepts, and asymptotes. (No conic section has all of these.)* <u>2) If you replace X with X-h, then move it right h units.</u> *That includes the center, vertices, foci, directex,, intercepts, and asymptotes.*

Section 5.1: The **base** of an **exponential function** $f(x)=a^x$ is a. They all take the same shape assuming the sign in front of the x is positive. If it is negative, it is a mirror image across the Y-axis. (Sloping down). The bigger the base, the quick it rises for x>0 (or falls for x<0 if it is a^{-x}) and the closer it is to the x-axis for x <0 (or for x>0 if it is a^{-x}). If a<0, then it is a mirror image across the x-axis. Translations are just like in 4.4. Adding a number moves it right, but replacing X with X + a number will move it left. The **natural base is e** which is about 2.718... It is used for continuous change like compounding interest or more realistically, the population increasing. If you need them, I will provide formulas you may need like $A = P\left(1+\frac{r}{n}\right)^{n*t}$ and $A=Pe^{rt}$.

Section 5.2: $f(x) = \log_a(x)$ is the **logarithmic function with base a**. *If* $y=\log_a(x)$ *then* $x = a^y$. *Important properties include* $\log_a(1) = 0$, $\log_a(a) = 1$, $\log_a(x) = \log_a(y)$ *means* $x = y$, $\log_a(a^x) = x$ *and* $a^{\log a(x)} = x$ *which means* a^x *is the inverse function of* $\log_a(x)$. The **one-to-one property** is the on $\log_a(x) = \log_a(y)$ *means* $x = y$. Because the log and exponent are inverses, their graphs are mirror images with the line x=y as the axis of symmetry. <u>If you write log(x) without a base, it is the **common log** which has 10 as the base. If the base is e, then it is the **natural log** and is written ln(x).</u> The common log and natural log have the same properties as log base a.

Section 5.3: **Changing the base** of a log can make it easier to get the answer. $\log_a(X) = \dfrac{\log_b(X)}{\log_b(a)}$

This is particularly useful for problems like $\log_{32}(16)$. Since both 16 and 32 are powers of 2, choose b to be 2. We get $\log_2(16)/\log_2(32) = 4/5$. The **product property, quotient property,** and **power property** apply to $\log_a$, log, and ln. Since it is easy to type, I will use the common log in the rest of this section, but you could use any of the logs. The easiest way to remember them is to reduce the operation in degree. For example, log(xy) has multiplication inside. Multiplication is repeated addition, so reducing the operation gives log(xy) = log(x) + log(y). Division is repeated subtraction, so log(x/y) = log(x)-log(y). Exponents are repeated multiplication so $\log(x^n) = n*\log(x)$.

Section 5.4: Solving exponent and logarithmic equations is just using the one-to-one property or the inverse property. <u>See "solving simple equations" on Page 408 for examples and a summary.</u> *Note that Example 1d was found by taking the ln of both sides. Examples 1e - 1g were found by raising both sides to the power of the base.*

Section 5.5. Know the **exponential growth model, exponential decay model, Gaussian model, logistic growth model, and logarithmic models.** They are $y = ae^{bx}$ (b>0), $y = ae^{-bx}$ (b>0), $y = ae^{-(x-b)^2/c}$, $y = a/(1+be^{-rx})$, and y = a +bln(x) respectively. *The first two are basically what we did in Section 5.1. The third one is the hill or normal distribution. The fourth one is the **sigmoidal curve**. The last one was in Section 5.2* <u>For the Gaussian and logistic growth models, I will just ask you to give the general shape because they do not give you precise instructions for plotting them.</u> Understand when we use each model.