

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I cannot give much extra time.

Show all work for all questions.

1) (2 points) What is the better known name of $\begin{bmatrix} 0 & \vdots & (\nabla f)^T \\ \dots & \dots & \dots \\ \nabla f & \vdots & \nabla^2 f \end{bmatrix}$?

2) (4 points) Prove that in general $|\bar{H}_1| < 0$.

3) (10 points) Prove that if $Y = F(X_1, X_2)$ is convex, then $F_{ii}'' \geq 0$ for all i .

4) (12 points) Find the degree of homogeneity for the function $Q(L, K) = 6L^{1/3}K^{1/2}$ using the formal proof. What does this tell you about the returns to scale of producing this item?

5) (16 points) For the equations $y = 3x^2 + 2xy + y^2 + 7$, find the first order total differentiation (dy) and the second order total differentiation (d^2y). What information does this give about the concavity of the function.

6) (16 points) If $f(x, y, z) = x^2 + 3xyz + 4y^2$, then find ∇f , \mathbf{H} , and \mathbf{H} .

7) (20 points) Find the bordered Hessian for $f(x, y) = 5xy$. Determine if f is quasi-concave, quasi-convex, or neither.

8) (20 points) Suppose $U(C, A) = 6A^{1/2}C^{1/3} + 6A^{1/3}C^{1/2}$. Find the slope of the indifference curve at the point (1, 64). Note that C is on the x -axis. Given your answer, how many apples would this person be willing to trade for a carrot? Briefly state how you reached that conclusion.