Wilfrid W. Csaplar Jr. Quantitative Methods... Review Sheet for Exam 1

This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

The review session will be at a time to be determined, probably Thursday 9/20 in the normal room.

Chapter 1: What are **exogenous variables, endogenous variables, parameters, relationships, equilibrium, statics,** and **dynamics**?

Chapter 2.1: Understand was **sets, subsets,** and **elements** are. Understand set notation and the symbols $\in$, $\subset$, $\supset$, $\subseteq$, $\supseteq$, $\not\subset$, $\not\supset$, $\not\subseteq$, $\not\supseteq$, and $\notin$. Know what $\Leftrightarrow$ means. Be able to draw and interpret **Venn diagrams**. What is the symbol for the **empty set**, a.k.a. **null set**? Know how to find the sets for $\cap$ and $\cup$. What are **disjoint sets**? What is the symbol for the **complement set**? Be able to find the **relative difference of two sets**. Be able to recognize a **partition of a set**.

Chapter 2.2: Now what the following sets are: **Z**, $\mathbf{Z}_+$, **Q**, $\Re$, $\Re_+$, and $\Re_{++}$. Know the following properties of real numbers: **closure, commutative laws for + and *, associate laws for + and *, distributive law, the properties of zero and one, negation, reciprocals, completeness, transitivity, reflexivity,** and **equality.** Knowing the **dimensions** (**units**) of economic variables will help you to write equations which make economic sense.

Chapter 2.3: Know how to plot an **ordered pair** on the **coordinate system (Cartesian Plain).** Know how to do a **Cartesian product** like $\{1, 2, 3\}\otimes\{4, 5, 6, 7\}$. Be able to intervals on a number line like, [1, 4), (-∞, 7], etc. Know the difference between an **open interval, closed interval**, and **half-open interval**. What do **bounded** and **compact** mean? Know how to tell if a set is **convex**. (A line connecting two points is entirely in the set.) Memorize the **Euclidian distance formula** (which is easier than it looks.) Know what an **ε-neighborhood** is. Note that the definitions of a **boundary point**, **interior point**, and **convex set** easier than they look.

Chapter 2.4: Know what **function, domain, range, image,** and **value** are. Be able to tell if the function is **one-to-one, onto,** and/or **one-to-one correspondence.** (The former is the vertical line test, while the middle one is the horizontal line test, and the latter is both tests.) When is a function **invertable**? Know what a **composite mapping** is. Know what is meant by **slope coefficient** and **intercept term** in a **linear function**. An **implicit function** is basically f(x, y)=0. (y is a function of x, but it is not explicitly written out.) Know that **quadratic** functions have a maximum or minimum at $x = -b/2a$ where $y = ax^2 + bx + c$. **Rectangular hyperbolas** are of the form $xy=\alpha$.. Know how to plot **power functions** ($y = ax^b$) and **exponential functions** ($y=a*b^x$). Know what **logarithmic functions** are including the **natural log**. Note that for all bases (including e so it applies to the natural log ln) that $\log_b(x*y)= \log_b(x)+\log_b(y)$ which means $\log_b(x^a) = a*\log_b(x)$ and $\log_b(x/y) = \log_b(x)-\log_b(y)$. The definitions of **concave** and **convex** are fairly simple (concave looks like a cave so all points on the **secant** are below the line.). Putting **strictly** in front of them means the equality does not hold. For **quasi-concave**, the **better set** of the isobar is convex. For **quasi-convex**, the **worse set** of the isobar is convex. Note that the **Cobb-Douglas** function $f(x,y)=x^a y^b$ normally uses the assumption that a+b<1.

Chapter 3.1-3.2: A **sequence** is a succession of numbers of the form f(1), f(2), f(3), . . . like $f(n) = n^2$ where $n \in Z_+$ or $a_1, a_2, a_3, \ldots$ Know how to tell if it has a **limit**. (For a n >N, $|a_n – L| <\varepsilon$ for an arbitrarily small ε.) If there is no limit, it is **divergent**. **Definitely divergent** if the limit is ∞ or -∞. It is **bounded** if there is a range the values do not exceed for large n. ($|a_n|<K$).

Chapter 3.3: Understand why $PV=FV_t/(1+r/n)^{nt}$ for **discrete compounding** and $FV_t = PV*e^{rt}$ or $PV = FV_t e^{-rt}$ for **continuous compounding**. There will be more of this in Chapter 3.5.

Chapter 3.4: The limits of sequences have nine properties which are very intuitive on Pages 79 and 81.

Monotonically increasing and **monotonically decreasing** are exactly what you would expect. It is **bounded** if there is both an upper bound and a lower bound.

Chapter 3.5: A **series** is a special type of sequence which is a summation. The standard notation is s_n where a sequence is a_n. If the $s_n = \sum a_t$ and $\lim_{n \to \infty} |a_{n+a}/a_n| = L$, then if $L < 1$ the series converges, if $L > 1$, the series diverges, and if $L = 1$, either could occur. (Check out $a_n = 0$, $a_n = 1$, and $a_n = (-1)^n$. What are s_n and $|a_{n+1}/a_n|$?) The **geometric series** is $s_n \sum a\rho^t$ converges if $|\rho|<1$ and $\lim_{n \to \infty} s_n = a/(1-\rho)$. This can be used to prove that PV = FV $_\infty$/r and PV of an payment yearly for n years is $PV = FV_t \left(\dfrac{1 - \left(\dfrac{1}{1+r} \right)^n}{1 - \dfrac{1}{1+r}} \right)$.

The internal rate of return is the value of r which makes the equation equal.

Chapter 5.1 - 5.3: Know what **marginal analysis** means. What is a **tangent line** and how does that relate to the slope of the line? Note that the limit of the slope of the **secant line** as Δx approaches zero is the slope of the tangent line. Know the definitions of **derivative** and **total differential**. Note that marginal cost is the derivative of total cost function. A function is **differentiable** at a point if the function is continuous at a point and if the **left-hand derivative = the right-hand derivative.** Basically, if there is an angle rather than a gentle bend, it is not differentiable at the angle.

Chapter 5.4: Rules for differentiation. Remember these rules $f(X) = c$, then $f'(X) = 0$. If $f(X) = mX$ then $f'(X)=m$. If $f(X) = X^n$ then $f'(X) = nX^{n-1}$. If $g(X) = c*f(X)$ then $g'(X) = c*f'(X)$. If $h(X) = g(X) + f(X)$ then $h'(X) = g'(X) + f'(X)$ and applies to adding even more terms. If $h(X) = g(X)*f(X)$ then $h'(X) = f(X)*g(X) + g'(X)*f(X)$ which can also be used with division by defining $k(X) = 1/f(X)$ or you can use the division rule. If $h(X) = f(X)/g(X)$ then $h'(X) = [f'(X)*g(X) - g'(X)*f(X)]/[g(X)]^2$. If you have $y = f(U)$ and $U = g(X)$ the $h(X) = f(g(X))$ and $h'(X)=f'(U)*g'(X)$. If the inverse of $Y = f(X)$ is $X = g(Y)$, then $g'(Y)=1/f'(X)$. If $f(X) = e^x$ then $f'(X) = e^x$. If $f(X) = \ln(X)$ then $f'(X) = 1/X$.

Chapter 5.5: Know the symbols for **second derivative or third** etc. A function is convex if $f'' \geq 0$ and it is strictly convex if $f''(X) > 0$ at all points or all but one point. Reverse the inequalities for concave and strictly concave. Note that the second derivative test tells us if we found a maximum or a minimum.

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1) For each of these functions, find the first derivative, second derivative, and third derivative. Find where the first derivative equal to zero. Is that point a maximum, minimum, or can't you tell? Show all work and briefly explain your logic.
A) $TC = 2500 - 500Q + 5Q^2$
B) $TPL = 10L^{1/2} - L$
C) $\Pi = P(Q)*Q - C(Q)$ where $P(Q) = 45Q + 1000*Q^{-1}$ and $C(Q) = 10 + 5Q$.

2) If the demand curve is given by $P(Q) = a - bQ$ then find the total revenue and marginal revenue functions. Show all work and briefly explain what you did.

3) If the demand curve is given by $P(Q) = b/Q$ then find the total revenue and marginal revenue functions. Show all work and briefly explain what you did. What does this function imply about the quantity this firm would want to produce?

4) If $\Pi = P(Q)*Q - C(Q)$ then find the following functions, marginal profit, marginal cost, and marginal revenue. Set marginal profit equal to zero. What does this mean about marginal revenue and marginal costs?