

Write your name on the cover of the test booklet and nowhere else. Failure to follow these directions will cost you 1 point. The test has 150 points (to be scaled up to 200 points) and is scheduled to take 75 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I will allow some extra time, but I will not allow much.

**Show all work on all questions.**

1) (8 points) Do EITHER Part A OR Part B.

A) Suppose the Cournot firm’s best response function is given by  $Q_1 = 30 - \frac{1}{2}Q_2$ , and their profit is given by  $\Pi_1 = -Q_1^2 - 3Q_1Q_2 + 300Q_1 - 20$ . Set up the Lagrangian for the Von Stackelberg leader and state how you found it.

B) Suppose we wanted to minimize your income (so you can work as little as possible) subject to your utility is  $U = 50$ . Your utility function is  $U(C, D) = C^{1/3}D^{1/2}$ , the price of a dog is \$15/dog, and the price of a cat is \$21/cat. Set up the Lagrangian and explain how you got it. **Do not solve it.**

2) (10 points) Do EITHER Part A OR Part B.

A) Prove that profit maximization will yield  $MR = MC$  using calculus.

B) Is the  $U(X, Y) = 12X^{1/3}Y^{1/3}$  a legitimate utility function? Prove your answer is true using the formal proof.

3) (24 points) Find all Nash equilibria in the following matrix. Prove that you found all and prove they are Nash equilibria. Does either firm have a dominant strategy? Find the cooperative equilibrium. Explain how you found it. What are the two players’ secure strategies? How did you find them?

Payoff Matrix		Pirates		
		High price	Medium Price	Low Price
Red Sox	High Price	22 36	23 31	14 45
	Low Price	27 38	21 20	26 44

**Answer THREE of the following four questions. Each are 36 points.**

4) Suppose that labor costs \$8/hour and capital costs \$12/unit and the firm’s production function is given by  $Q=K^{1/2}L^{1/3}$ . Derive the total cost function which minimizes costs for a given output  $Q$ . Do NOT worry about  $\lambda$ . What are the marginal cost and average total cost functions? Show all work.

5) Suppose your utility function is  $U(B, C) = B^{1/2}C^{1/4}$ . The price of a ball is \$4 each and the price of a cap is \$8 each. Find the utility maximizing consumption of balls and caps if you’re your income is \$384. How much is your utility? Show all work. Approximately, how much is the marginal utility of another dollar of income? If your income went up \$32, approximately how much would your utility go up?

6) Suppose your utility function is given by  $U(D, M, H) = D^{1/3}M^{1/3}H^{1/3}$ . The price of a dinner (D) \$20/meal, price of a movie (M) is \$10/ticket, and a hat (H) costs \$40/hat. Your income is \$400. You also have a time constraint of a dinner takes 3 hours while a movie takes 1.5 hours. You have 24 hours of time to spend. Find the utility maximizing levels of D, M, and H. Show all work. Do **not** worry about finding  $\lambda$ .

7) Suppose that both firms are facing the following demand and total cost functions:  $P = 1400 - 2(Q_i)$  and  $TC_i=100 + 200Q_i + 3Q_i^2$ . Use this to derive the best response function for Firm 1 and the equilibrium output for each firm, assuming that the firms are Cournot style firms. What are the outputs of the two firms and the market price? Show all work.

