Place your name on the back of this sheet of paper and nowhere else. Staple your answers on the front of this sheet of paper. Failure to follow these directions will cost you 1 point. If you use double-sided printing or print on the back of scrap paper, I will give you one additional point.

## Show all work on all questions.

1) ( 25 points) Suppose your utility function is given by $U(C, M, H)=16 C^{1 / 4} \mathrm{M}^{1 / 4} \mathrm{H}^{1 / 4}$ where C is the number of concerts, M is the number of movies, and H is the number of hats. The price of a concert is $\$ 50 /$ concert, the price of a movie is $\$ 25 /$ movie, and the price of a hat is $\$ 50$. Your income is $\$ 300$. Concerts take 3 hours while movies take 1.5 hours and hats take no time. You have 12 hours you can spend on entertainment. Set up the Lagrangian for this utility maximization stating how you found it. Use it to find the utility maximizing quantity of hats, concerts, and movies. Approximately, how much would your utility increase if your income went up \$4? How did you get that? Show all work.
2) (10 points each) For each of the following, determine if it is a valid utility function. If you do the "returns to scale" test, do the formal test - not the quick test.
A) $U(A, B)=3 A^{2} B^{2}$
B) $U(C . D)=3 C^{2 / 3} D^{2 / 3}$
C) $U(E, F)=3 E^{2 / 3}+3 F^{2 / 3}$
E) $U(G, H)=\ln \left(G^{*} H\right)$ Do not worry about the "returns to scale" test.
3) (10 points) For utility function $U(J, K)=6 J^{1 / 3} K^{1 / 2}$, what transformation, $F(U)$, would you do to make the problem easier to deal with? Prove that $\mathrm{F}(\mathrm{U})$ is a valid transformation and find $\mathrm{F}(\mathrm{U})$.
4) (15 points) Suppose your production function is given by $\mathrm{Q}=4 \mathrm{~K}^{1 / 2} \mathrm{~L}^{1 / 4}$. Labor costs $\$ 10 /$ hour while capital costs $\$ 5 /$ hour. Find the cost minimizing capital and labor if you want to produce 216 items Approximately how much more would it cost you if you wanted to produce 2 more?. Show all work.
