

Write your name on the cover of the test booklet and nowhere else. Failure to follow these directions will cost you 1 point. The test has 150 points (to be scaled up to 200 points) and is scheduled to take 75 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I will allow some extra time, but I will not allow much.

Show all work on all questions.

1) (8 points) Do EITHER Part A OR Part B.

A) Suppose your income this year was \$500, next three years was \$600, \$700, and \$800 respectively. Your utility function is given by $U = C_0^{1/5} C_1^{1/5} C_2^{1/5} C_3^{1/5} C_4^{1/5}$. Set up the Lagrangian and state how you got it. **Do not solve it.**

B) Suppose we wanted to minimize your income (so you can work as little as possible) subject to your utility is $U = 100$. Your utility function is $U(C, M) = C^{1/3} M^{1/2}$, the price of a concert is \$33/show, and the price of a movie is \$8/ticket. Set up the Lagrangian and explain how you got it. **Do not solve it.**

2) (10 points) Do EITHER Part A OR Part B.

A) Is the $U(X, Y) = 8X^{1/4} Y^{1/4}$ a legitimate utility function? Prove your answer is true using the formal proof.

B) What are **two** of the tests for a legitimate utility function? Explain the economics as to why we have that test.

3) (24 points) Find all Nash equilibria in the following matrix, if any exist. Prove that you found all and prove they are Nash equilibria. Does either firm have a dominant strategy? How can you tell? Find the cooperative equilibrium. Explain how you found it. What are the two players' secure strategies? How did you find them?

Payoff Matrix		Pirates		
		High price	Medium Price	Low Price
Red Sox	High Price	6 15	7 8	5 11
	Low Price	9 14	3 4	10 12

4) (30 points) Suppose your utility function is $U(H, C) = H^{1/4} C^{1/2}$. The price of a hat is \$30/hat and the price of a coat is \$60/coat. Find the utility maximizing consumption of hats and coats if you're your budget is \$1440. How much is your utility? Show all work. Do not worry about λ .

5) (36 points) Answer EITHER Part A OR Part B.

A) Suppose that two firms are facing the following demand and total cost functions: $P = 115 - 5(Q_1)$ and $TC_i = 40 + 15Q_i + 5Q_i^2$. Use this to derive the best response function for Firm 1 and the equilibrium output for each firm, assuming that the firms are Cournot style firms. What are the outputs of the two firms and the market price? Show all work.

B) Suppose the Cournot firm's best response function is given by $Q_1 = 40 - \frac{1}{2}Q_2$, and the leader's profit is given by $\Pi_L = -Q_L^2 - 3Q_L Q_F + 60Q_L - 23$. Set up the Lagrangian for the Von Stackelberg leader and state how you found it. Find the equilibrium outputs for the two firms and the price sold. The industry demand is $P = 10 - 2Q_1$

6) (42 points) Answer EITHER Part A OR Part B.

A) Suppose that labor costs \$10/hour and capital costs \$20/unit and the firm's production function is given by $Q = K^{1/4} L^{1/2}$. Derive the total cost function which minimizes costs for a given output Q . **How much is λ ?** What are the total cost, marginal cost and average total cost functions? Show all work and verify that $\lambda = MC$.

B) Suppose your utility function is given by $U(D, M, H) = 5D^{1/5} M^{2/5} H^{1/5}$. The price of a dinner (D) \$20/meal, price of a movie (M) is \$20/ticket, and a hat (H) costs \$10/hat. Your income is \$200. You also have a time constraint of a dinner takes 2 hours while a movie also takes 2 hours. You have 15 hours of time to spend. **Transform the utility function to something easier to use** and prove that your transformation is a valid transformation. Find the utility maximizing levels of D, M, and H. Show all work.