Wilfrid W. Csaplar Jr., Ph.D. Mathematics 103 Final 2020/11/24
Do NOT write your name anywhere. (Canvas will tell me who turned in the exam.) Take pictures of your answers and use your own software or https://pdfcandy.com/ to create a PDF for each answer which requires an upload. If it is large, resize it to A4. Upload that to Canvas. Upload each answer as a separate file with that question. Failure to follow directions will cost you one point. People with Apple products may need to us CamScanner app.

You are not allowed to use your books, notes, the internet, or other people when taking this test. You can use the internet to access Canvas and to convert your answers to PDF files. Nothing else.

If you run out of time or lose your internet connection, you can do a second submission. You do NOT have to redo the questions you already did. I will be able to see every submission. If you have problems, you can always contact me via Zoom or e-mail. If you use Zoom, open it in a new tab or window.

Failure to follow these directions will cost you 1 point. The test has 240 points (to be scaled down to 150 points) and is scheduled to take 120 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I have it set up to only give you an hour and a half.

## Show all work on all questions.

1) (12 points) Answer EITHER Part A OR Part B.
A) Solve this system in any manner you want. $3 \mathrm{X}+2 \mathrm{Y}=2,6 \mathrm{X}+6 \mathrm{Y}=5$.
B) Solve this system in any manner you want. $2 \mathrm{X}-5 \mathrm{Y}=0,4 \mathrm{X}+10 \mathrm{Y}=4$.
2) (12 points) Answer EITHER Part A OR Part B.
A) Solve this system in any manner you want. $4 \mathrm{X}+3 \mathrm{Y}=3,-2 \mathrm{X}+6 \mathrm{Y}=1$.
B) Solve this system in any manner you want. $10 \mathrm{X}-3 \mathrm{Y}=4,-20 \mathrm{X}+9 \mathrm{Y}=-7$.
3) (14 points) Answer EITHER Part A OR Part B.
A) Use synthetic division with $X=2$ and $P(X)=X^{3}+10 X^{2}+19 X-30$. What two things does that answer tell you? Briefly explain how you reached those conclusions.
B) Use synthetic division with $X=-2$ and $P(X)=X^{3}-6 X^{2}+5 X+12$. What two things does that answer tell you? Briefly explain how you reached those conclusions.
4) (14 points) For EITHER the equation in Part A OR the equation in Part B, list all possible rational roots. Use Descartes' Rules of Signs to find out the number of possible rational positive roots and the number of possible negative rational roots. Do NOT actually find the roots.
A) $F(X)=6 X^{4}+X^{3}+3 X^{2}-2 X+14$
B) $F(X)=10 X^{4}+2 X^{3}-7 X^{2}+7 X+15$
5) (16 points) Answer EITHER Part A OR Part B.
A) Do $\left(2 \mathrm{X}^{3}-7 \mathrm{X}+2\right) /(\mathrm{X}+2)$ using long division.
B) $\operatorname{Do}\left(2 \mathrm{X}^{3}-17 \mathrm{X}+3\right) /(\mathrm{X}+3)$ using long division
6) (20 points) Answer EITHER Part A OR Part B.
A) You have coffee which costs $\$ 3 / \mathrm{lb}$ and other coffee which costs $\$ 8 / \mathrm{lb}$. You want to mix 9 lb of coffee which costs $\$ 5 / \mathrm{lb}$. How much of each do you need? After you setup the equations, you can solve it any way you want.
B) You have a $10 \%$ salt solution and a $40 \%$ salt solution. You want 9 liters of a $30 \%$ solution. How much of each do you need? After you setup the equations, you can solve it any way you want.
7) (22 points) Answer EITHER Part A OR Part B.
A) Use Gaussian elimination to get the system into triangular form. Then find the solution.
$\mathrm{X}-\mathrm{Y}=10, \quad 3 \mathrm{X}+2 \mathrm{Z}=20, \quad 6 \mathrm{X}-3 \mathrm{Y}+2 \mathrm{Z}=60$
B) Use Gaussian elimination to get the system into triangular form. Then find the solution.
$\mathrm{X}-\mathrm{Z}=20, \quad 2 \mathrm{X}+\mathrm{Y}=10, \quad 5 \mathrm{X}+\mathrm{Y}-3 \mathrm{Z}=60$
8) (22 points) Answer EITHER Part A OR Part B.
A) List all possible rational roots. Use Descartes' Rules of Signs to find out the number of possible rational positive roots and the number of possible negative rational roots. Use that information and synthetic division to find all of the rational zeros and factor it.
$\mathrm{P}(\mathrm{X})=8 \mathrm{X}^{3}-12 \mathrm{X}^{2}+6 \mathrm{X}-1$
B) List all possible rational roots. Use Descartes' Rules of Signs to find out the number of possible rational positive roots and the number of possible negative rational roots. Use that information and synthetic division to find all of the rational zeros and factor it.
$P(X)=X^{3}+9 X^{2}+27 X+27$
9) ( 26 points) Answer EITHER Part A OR Part B. Hint for both parts: after you get it setup in a form which you can solve it, one of the zeros is $\mathrm{Q}=-1$.
A) Suppose the cost function is given by $C(Q)=Q^{3}-13 Q+150$. What range of outputs could you produce and still have your costs less than $\$ 162$ ?
B) Suppose the price is given by $P(Q)=Q^{3}-2 Q^{2}-13 Q+100$. What range of outputs could you produce and still have your costs less than $\$ 110$ ?
10) ( 26 points) Answer EITHER Part A OR Part B.
A) Use Gaussian elimination to get the system into triangular form. Then find the solution. $3 \mathrm{X}-2 \mathrm{Y}+\mathrm{Z}=20, \quad-\mathrm{X}+2 \mathrm{Y}-\mathrm{Z}=0, \quad 4 \mathrm{X}+3 \mathrm{Y}=70$.
B) Use Gaussian elimination to get the system into triangular form. Then find the solution. $2 \mathrm{X}-2 \mathrm{Y}+\mathrm{Z}=10, \quad-\mathrm{X}+2 \mathrm{Y}=10,3 \mathrm{X}+4 \mathrm{Y}+\mathrm{Z}=80$.
11) (28 points) For EITHER the equation in Part A OR the equation in Part B. Find all the zeros any way you want. Show all work including failed attempts. They may get you more partial credit. Hint: Doing Descartes' Rule of Signs will help. Remember what must be true if $F(A)$ and $F(B)$ have different signs. You know $\mathrm{F}(0)$.
A) $F(X)=18 X^{3}+33 X^{2}+20 X+4$
B) $F(X)=8 X^{3}+28 X^{2}+30 X+9$
12) (30 points) Answer EITHER Part A OR Part B.
A) Use Gaussian elimination to get the system into triangular form. Then find the solution. $3 \mathrm{X}+2 \mathrm{Y}+\mathrm{Z}=50, \quad 2 \mathrm{X}-\mathrm{Y}=10, \quad \mathrm{X}+3 \mathrm{Y}+\mathrm{Z}=40$
B) Use Gaussian elimination to get the system into triangular form. Then find the solution.
$6 \mathrm{X}-\mathrm{Y}+\mathrm{Z}=50, \quad 2 \mathrm{X}+2 \mathrm{Y}=40, \quad 4 \mathrm{X}-3 \mathrm{Y}+\mathrm{Z}=10$
