Place your name on the back of this sheet of paper and nowhere else. Staple your answers face up on the front of this sheet of paper. Failure to follow these directions will cost you 10 points. If you use double-sided printing or print on the back of scrap paper, I will give you one additional point.

## Show all work and briefly explain all answers.

1) (35 points) Suppose your utility function for bathrobes, jackets, and pencils, is given by $\mathrm{U}(\mathrm{B}, \mathrm{J}, \mathrm{P})=12 \mathrm{~B}^{1 / 3} \mathrm{~J}^{1 / 6} \mathrm{P}^{1 / 6}$. The price of a bathrobe is $\$ 10 /$ robe. The price of a jacket is $\$ 10 / \mathrm{jacket}$. And the price of a pencil is $\$ 5 /$ pencil. Your income is $\$ 160$. Setup the budget constraint. Setup a Lagrangian and use it to find the optimal number of each item.
2) (15 points each) For each of the following proposed utility functions, do all three tests to determine if it is a valid utility function. For the returns to scale, you must do the formal test. State your conclusion and how you reached it.
A) $U(F, B)=12 F^{2} B^{2}$
B) $\mathrm{U}(\mathrm{V}, \mathrm{W})=6 \mathrm{~V}^{2 / 3} \mathrm{~W}^{2 / 3}$
C) $U(E, Q)=6 E^{2 / 3}+6 Q^{2 / 3}$
3) (10 points each) For each of the following, find a transformation which will help you do the maximization. Prove that your transformation is a valid transformation and find what $F(U)$ equals.
A) $U(M, P)=4 M^{1 / 4} \mathrm{P}^{1 / 2}$
B) $U(X, Z)=12 X^{1 / 2} Z^{1 / 3}$
