

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but I will not give much.

Show all work on all questions.

1) (4 points) Answer EITHER Part A OR Part B.

A) When we do row operations for a system of equations, are we using the *elimination method* or the *substitution method* of solving equations? **Briefly** explain your logic.

B) Is it possible for a homogeneous system of equations to be overdetermined? **Briefly** explain your logic.

2) (10 points each) Use the following matrices to answer FOUR of the following parts.

$$A = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 5 & -3 \end{bmatrix}, D = \begin{bmatrix} 4 & -5 \\ -4 & 10 \end{bmatrix}$$

A) trace(AB)

B) $3C - D$

C) $A^T C$

D) $A^T B^T$ and BA

E) $2B + 3A^T$

F) trace(C + D)

3) (12 points) Solve the system of equations using any method you choose. (These equations are respectively for the IS curve and the LM curve of the IS/LM diagram in *Intermediate Macroeconomics*.)
 $r = 65 - .05Y$, $r = 5 + .01Y$.

4) (14 points) Answer EITHER Part A OR Part B.

A) Setup the following equations in matrix form. Show all work. Do **not** solve the system.

$$Q_1^D = 200 - 4P_1 + P_2 - 2P_3 \quad Q_1^S = 25P_1 - 5 \quad Q_2^D = 150 + 2P_1 - 3P_2 \quad Q_2^S = 6P_2 - 20$$

$$Q_3^D = 95 - P_1 - 2P_3 \quad Q_3^S = P_3 - 30$$

B) Estimate the solution to the following system of equations using the **graphing** method.

$$Q^D = 10 - \frac{1}{2}P$$

$$Q^S = 2P - 6$$

5) (14 points) Answer EITHER Part A OR Part B referring to the following system of equations.

$$X + 3Y - Z = 20, \quad 2X - 2Y + Z = 5, \quad 3X + Y = 15.$$

A) Determine how many linearly independent equations there are in the system of equations.

B) Solve the system of equations. If it is not solvable, then explain why it cannot be solved.

6) (16 points) Answer EITHER Part A OR Part B. This question at first appears to be difficult, but it in reality neither part is not as hard as they appear if you take things step by step.

A) Suppose the population matrix $P = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}$ and suppose the population of the two cities totals 100 people. What distribution of the people would result in the populations of the two cities being the same every year? In other words, what values of X and Y would result in $\begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$ and $X+Y = 100$. Hint: Do the multiplication out and then solve the system of equations.

B) Find the value of X such that $\begin{bmatrix} 4 & 2 \\ -6 & X \end{bmatrix}$ is idempotent. Hint: Take the definition of idempotent, do the multiplication out and you will get three equations involving X. Any of the three equations by itself will could be solved for X. Use one of the two easier equations to solve for X. I suggest (but do not require) that you verify your solution using another equation.