Wilfrid W. Csaplar Jr. Quantitative Methods... Review Sheet for Exam 2

This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

The review session's time will be determined, probably 3:30 on Thurs. 3/6 in the normal room.

Chapter 5.4: **Rules for differentiation**. Remember these rules f(X) = c, then f'(X) = 0. If f(X) = mX then f'(X)=m. If f(X) = X^n then f'(X) = nX^{n-1}. If g(X) = c*f(X) then g'(X) = c*f'(X). If h(X) = g(X) + f(X) then h'(X) = g'(X) +f'(X) and applies to adding even more terms. If h(X) = g(X)*f(X) then h'(X) = f(X)*g(X) + g'(X)*f(X) which can also be used with division by defining k(X) = 1/f(X) or you can use the division rule. If h(X) = f(X)/g(X) then h'(X) = [f'(X)*g(X) – g'(X)*f(X)]/[g(X)]2. If you have y = f(U) and U = g(X) the h(X) = f(g(X)) and h'(X)=f'(U)*g'(X). If the inverse of Y = f(X) is X = g(Y), then g'(Y)=1/f'(X). If f(X) = e^x then f'(X) = e^x. If f(X) = ln(X) then f'(X) = 1/X.

Chapter 5.5: Know the symbols for **second derivative** or third etc. A function is **convex** if f''(X) $\geq$ 0 and it is **strictly convex** if f''(X) >0 at all points or all but one point. Reverse the inequalities for **concave** and **strictly concave**. Note that the second derivative test tells us if we found a maximum or a minimum.

Section 6.1: What are meant by **extreme values, unconstrained, constrained, global maximum, local maximum, global minimum, local** and **minimum**? The difference between global and local maximum and minimum are global is for all x, while local is for $\hat{x} - \varepsilon \leq x \leq \hat{x} + \varepsilon$. **First order condition** is f'(x*) =0 is a necessary condition for a local maximum, local minimum, and an **inflection point** (because it gives a **stationary point**) but not a sufficient condition. Note that most of this section is just analyzing different examples. Understand what $f'(x) \neq 0, \forall x$ means. (No maximum and no minimum.)

Section 6.2: **Second-order conditions** can determine if it is a maximum or minimum. f''(x*)<0 is maximum and f''(x*) > 0 is a minimum. f''(x*) = 0 could be either of those or an inflection point. Do not worry about the Taylor Series. Like Section 6.1, much of this chapter is just examples.

Section 6.3: Optimization **over an interval** is just like the local optimization except you have two differences. *First, check to make sure any extreme points you find are actually in the interval.* For example, finding the minimum of $f(x) = x^2 + 2x$, for $x \in [0, 100]$ will give a minimum at x = -1, which is not in the interval. *Second, check the limits of the interval*. In other words, check f(0) and f(100) in this example. If the maximum or minimum is inside the interval, then it is an **interior solution** and if it is not, then it is a **corner solution**. If there is a **constraint** that says $x \leq L$ or $x \geq L$, then when L is a **binding constraint**, you get a corner solution. The **shadow price of the constraint L** is the value of $f'(L)$.

Non-graded Assignment #4A to be reviewed with Assignment #4.
Show all work on all questions.

1) (15 points) If a firm's demand curve is given by $P = 112 - 2Q$ and its cost function is given by $TC = 3Q^2 + 2Q + 10$. Find their profit function. Find the profit maximizing output. Use the second derivative test to prove it is a maximum not a minimum.

2) (20 points) If a firm's demand curve is given by $P = 222 - 4Q$ and its cost function is given by $TC = Q^2 + 2Q + 10$. Find their profit function. Suppose the building is only big enough to produce 20. Find the profit maximizing output. Use the second derivative test to prove it is a maximum not a minimum. What is the profit at the maximum? What is the shadow price of the constraint? Approximately how much are profits decreased by the small size of the building?

3) (15 points) Find the maximum and minimum of the function $Y = X^2 + 8X + 10$ over the range $X \in [0, 100]$.

4) (15 points) Suppose a short-run profit function is given by $30X - 3X^2 - 85$. Maximize profits subject to $0 \leq X$. In the long-run, what should this firm do? Show all work and explain your logic.

5) (20 points) Suppose the demand function is given by $Q_D = 11 - P_C$ and the supply function is given by $Q_S = -4 + \frac{1}{2}P_S$. The tax rate is represented by t. What is the equation relating the two prices? What is the equation which says we are in equilibrium? Find the equation which determines the quantity produced at equilibrium as a function of t. What is the function for total tax revenue? Find the output which maximizes tax revenue. Prove it is a maximum not a minimum. For each step, show all work and briefly explain what you did.