

Place your name on the back of this sheet of paper and nowhere else. Staple your answers face up on the front of this sheet of paper. Failure to follow these directions will cost you 1 point. If you use double-sided printing or print on the back of scrap paper, I will give you one additional point.

**Show all work for all questions.**

1) (10 points) Prove that  $\begin{bmatrix} 3/4 & 3/4 \\ 1/4 & 1/4 \end{bmatrix}$  is idempotent.

2) (10 points) For the matrices  $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 9 & 0 \\ 8 & 5 & 7 \end{bmatrix}$ ,  $B = \begin{bmatrix} 15 & 12 & -2 \\ 6 & -3 & 10 \\ -1 & 11 & 13 \end{bmatrix}$ , find  $A^T + B^T$  and show that  $(A+B)^T$

3) (15 points) For the matrices  $A = \begin{bmatrix} 2 & 4 & 6 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ , prove that  $\text{Trace}(AB) = \text{Trace}(BA)$

4) (15 points) If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ , then find  $A^{-1}$ . Prove that  $A^{-1}A = I$ .

5) (5 points) Prove the matrix in Question #1 is singular.

6) (15 points) Find a value for  $x$  such that the matrix  $\begin{bmatrix} 1/2 & 1/3 \\ x & 1/2 \end{bmatrix}$  is idempotent. Hint: repeat your process for Question #1, find the value for  $x$ .

7) (5 points) We will later define  $I_n$  as the  $n \times n$  identity matrix. What is the  $\text{Trace}(I_n)$ ? How do you know?

8) (10 points) Suppose that  $A$  is an  $m \times n$  matrix and  $B$  is an  $n \times m$  matrix. Prove that  $(AB)^T \neq B^T A^T$ .

9) (15 points) If  $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ , then find  $A^{-1}$ . Prove that  $A^{-1}A = I$ .

