Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 150 points (to be scaled up to 210 points) and is scheduled to take 75 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but I will not give much.

Show all work for all questions.

1) (10 points) Solve the following system of equations by the substitution method.
$3X + Y = 10$ $2X + 2Y = 8$

2) (10 points) Find the second derivative of ONE of the following functions to determine if it is strictly concave, concave, convex, strictly convex, none of the above. State how you can tell. Assume the domain is $\Re_+$.

A) $F(X) = e^{X^2}$

B) $F(X) = \ln(X^2)$

3) (10 points) Answer EITHER Part A OR Part B.
A) Find the slope of the marginal revenue function if the total revenue function is given by:
$TR = -5Q^2 + 240Q$.
B) Suppose the total cost function as a function of price is $TC(P) = 200 - 3P$. What is the change in the costs caused by raising your price by one unit? Does TC increase or decrease as P increases? What is the economic reason for that change?

4) (10 points) Answer EITHER Part A OR Part B.

A) In ECON 360, we have the formula, $g = \dfrac{t - a_i t_i}{1 - a_i}$. Pretend that t and t_i are numbers so that you can treat them like constants. Find dg/da_i using the quotient rule. Use this to prove that an increase in a_i does not always increase g even though the book says it does.
B) Suppose $F(X) = G(X)/H(X)$. Obviously, $F(X) = G(X)*(H(X))^{-1}$ Use the quotient rule to take the derivative of the first equation and use the product and chain rules for the derivative of the second equation. Prove you get the same answer either way.

5) (20 points) Solve the following system of equations using the any method you want.
$Q_1^D = 100 - 10P_1 + 5P_2$ $Q_1^S = 70P_1 - 40$ $Q_2^D = 224 + 4P_1 - 8P_2$ $Q_2^S = 72P_2 - 88$

6) (20 points) For EITHER Part A OR Part B, write the equations in matrix form.. Use row operations to get the matrix into reduced row-echelon form if possible and solve it. <u>If it is not possible</u> to get it into reduced row-echelon form, then go as far as you can and explain what the problem is. How do you know the equations have the problem you say they have?
A) $2X + Z = 10$ $X + Y = 6$ $7X + 3Y + 2Z = 38$
B) $X - 3Z = 4$ $X + Y = 5$ $5X + 2Y - 9Z = 14$

7) (22 points) Find all local maxima, local minima, and inflection points for ONE of the following equations. For each point, determine if it is a local maxima, local minima, or inflection point. Explain how you reached your conclusions.
A) $F(X) = X^3 - 9X^2 + 27X + 3$
B) $F(X) = X^3 - 12X^2 + 45X + 6$

8) (22 points) Suppose a firm has an inverse demand function of P(Q) = 190-2Q and a total cost function of TC = Q^2 + 10Q + 5. Find the profit maximizing output. Use the second order conditions to prove it is profit maximizing.

9) (26 points) Do EITHER Part A OR Part B.
A) Suppose a monopoly has an inverse demand function of P = 140 - 4Q and a total cost function of TC = Q^2 + 10Q +10. Find the profit function. Find the profit-maximizing output if they are not legally allowed to produce more than 10. Prove that the global maximization of profit is that output. What is the shadow price of the constraint?
B) Suppose a firm's demand curve is given by Q = 306 -3P. Their total cost function is given by TC=2Q. Find their total revenue and total cost functions as a function of P. Find the profit-maximizing price. Suppose the monopoly is constrained to charge less than 30. What is the shadow price of the price ceiling? Approximately, how much could the profits increase if they could raise their price by 2?