Wilfrid W. Csaplar Jr., Ph.D. Business 205 Exam #3 2014/5/1

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 150 points (to be scaled up to 210 points) and is scheduled to take 75 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but I will not give much.

Show all work for all questions.

1) (10 points each) For the following matrices, do TWO of the following calculations.

$$A = \begin{bmatrix} 3 & 5 \\ 2 & -3 \\ 1 & -5 \end{bmatrix}, B = \begin{bmatrix} -3 & 4 & -2 \\ 0 & 6 & -1 \end{bmatrix}, C = \begin{bmatrix} 2 & 3 & -3 \\ 10 & -10 & 4 \end{bmatrix}$$

A) $A^T - 3B$
B) AC
C) $B^T - 2A$

2) (10 points) Answer EITHER Part A OR Part B.
A) Suppose you were working with the utility function $U(J, K, L) = 4J^{1/2}K^{1/3}L^{1/4}$. What positive monotonic transformation F(U) might you use to simplify this? Prove that your transformation is a legitimate one and then find F(J, K, L).
B) Explain why taking a positive monotonic transformation of $U(P, Y) = P^{1/2} + Y^{1/2}$ would not help.

3) (10 points) Answer EITHER Part A OR Part B.
A) Prove that $\begin{bmatrix} 1 & 0 \\ X & 0 \end{bmatrix}$ is idempotent for all X.
B) Find the trace of $\begin{bmatrix} 3 & 2 & 4 & 5 \\ 7 & 1 & 9 & 0 \\ -3 & -5 & -8 & 9 \\ 5 & 4 & 5 & 2 \end{bmatrix}$

4) (10 points) Suppose a utility function of carrots (C) and mangos (M) is given by $U(C,M) = 18C^{1/3}M^{1/2}$. Find ∇U and $\nabla^2 U$.

5) (10 points) Answer EITHER Part A OR Part B.
A) If the number of hats (H) you have at time t is given by $H(t) = 2t^{1/2}$ and the number of shirts (S) you have at time t is given by $S(t) = 20 - ½t$. If your utility function is given by $U(H, S) = 4H^{1/2}S^{1/2}$, then how much does your utility change over time?
B) If $F(X, Y, Z) = X^2 + 3XZ + Y^2 + Z^2 + 4YZ$, then prove that $F_{XZ}'' = F_{ZX}''$

6) (14 points) Answer EITHER Part A OR Part B.
A) Use Cramer's rule to prove that a homogenous system of equations will always have the trivial solution. To do this, you just need to find the value of the i^{th} variable.
B) Suppose that to produce \$1 worth of food (F), you require \$0.40 worth of food and \$0.20 worth of energy (E). Suppose that to produce \$1 worth of energy, you need \$0.30 worth of food and \$0.10 worth of energy. Set up the Leontief Input-Output matrix. Use that matrix to find the matrix which can be used to find out how much of each good you need to produce. State how you found that matrix. Set up the

equation A**x**=**b**, where A is the new matrix you found and assuming you want to sell \$50 worth of food and \$20 worth of energy. Do not worry about solving it.

7) (20 points) Answer EITHER Part A OR Part B.
A) Suppose $U(C, D) = 4(2C+3D)^{1/2}$ where C is coats and D is dresses. Find the first and second total differentiation. Given your answer, is the function strictly convex, convex, strictly concave, concave, or indeterminate? Explain your logic.
B) Suppose a production function is given by $Q(L, K, H) = L^{1/3}K^{/14}H^{1/4}$ where H is human capital. Use the implicit function theorem to find $\partial L/\partial H$ along an isoquant, assuming that K is constant. What does that mean about the substitution of L and H?

8) (26 points) Suppose that you have a system of equations Q = 250 - 5P and Q = 22P - 20. Write the equations in such a way that you can put them in matrix form. Write them in A**x**=**b** form. Find A^{-1}. Use A^{-1} to find the values of P and Q.

9) (30 points) Find A^{-1} for $A = \begin{bmatrix} 2 & 0 & 4 \\ -2 & 1 & 0 \\ 3 & 0 & -5 \end{bmatrix}$ using the minor, cofactor, adjoint method.