Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 240 points (to be scaled up to 250 points) and is scheduled to take 120 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but I will not give much.

Show all work for all questions.

1) (8 points) Answer EITHER Part A OR Part B.
A) What is another way to write $(((A^T)^T)^{-1})^{-1}$? Explain your logic.
B) Prove that $(AB)^T \neq A^T B^T$.

2) (10 points) Find the degree of homogeneity of the utility function U(R, T) = 5RT.

3) (10 points) Answer EITHER Part A OR Part B.

A) Find $\begin{vmatrix} 3 & 2 & 0 \\ -2 & -3 & -1 \\ 0 & 1 & 0 \end{vmatrix}$ B) Find $\begin{vmatrix} 3 & 5 & 9 & 2 & 2 \\ 0 & 1 & 5 & 8 & 8 \\ 0 & 0 & 5 & 2 & 5 \\ 0 & 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 0 & 4 \end{vmatrix}$

4) (10 points) Answer EITHER Part A OR Part B.
A) Is the function $f(X) = \begin{cases} 2X - 4 & X < 2 \\ X - 2 & X \geq 2 \end{cases}$ differentiable over the whole range of (-∞, ∞)? Explain your logic.
B) If F(X) = G(X)*H(X), then what is F'(X)?

5) (14 points) Take the first two derivatives of EITHER the function in Part A OR the function in Part B and determine the slope and whether it is strictly convex, convex, strictly concave, concave, or none of the above. Explain your logic.
A) $F(X) = X^2 - 4X + 4$
B) $F(X) = -X^4$

6) (14 points) Answer EITHER Part A OR Part B.
A) Draw (-4, 8]. Which of the following describes the area, open interval, half-open interval, closed interval, bounded, and compact? (It could be one or more.) BRIEFLY state how you found each one?
B) Draw four points on **one** indifference curve for the utility function U(B, P) = BP, where B = boots and P = pens. State how you found the four points.

7) (14 points) Answer this question for EITHER the sequence in Part A OR the sequence in Part B. Find the first five terms of the sequence. Which of the following descriptions apply: has a limit, is bounded, is divergent, is definitely divergent, is monotonically increasing, is monotonically decreasing? (It could be more than one.) Briefly state your logic.
A) f(n) = (n+3)/n
B) $f(n) = (-1)^n$

8) (14 points) Answer EITHER Part A OR Part B.
A) We know that for a single payment FV_t, t years into the future, $PV = FV_t/(1+r)^t$. This is a sequence. For an infinitely lived bond which makes annual payments of \$100 per year, find the value of the bond. This question is asking you to find the limit of the geometric series $s_n = \sum a\rho^t$. You need to tell me what a and ρ are and find the $\lim_{n\to\infty}(s_n)$.
B) Suppose a bond has a coupon rate of 6% with coupon payments made annually. The face value of the bond is \$1000. It matures in 10 years. If the current interest rate is 8%, then use the formula to find the present value. Set up the equation briefly stating why you chose each value for each variable.

$$PV = FV_t\left(\frac{1-\left(\frac{1}{1+r}\right)^n}{1-\frac{1}{1+r}}\right).$$

9) (16 points) Answer EITHER Part A OR Part B.
A) Suppose the supply curve is given by $Q = 40P$ and demand is given by $Q = 500 - 10P$. Write this in matrix form. Use Cramer's rule to solve the equations.
B) Use the minor, cofactor, adjoint method of inverting a matrix to prove the short cut we learned for the determinant of a 2x2 matrix actually works. Briefly explain what you are doing.

10) (16 points) Answer EITHER Part A OR Part B.
A) If the production function is given by $Q(L, K) = 6K^{1/3}L^{1/2}$, then find the MPK, slope of the MPK, and how much the MPK changes when L changes.
B) If $U(G, H) = 4G^{1/2}H^{1/2}$ and the number of gloves you have at time t is given by $G(t) = 1 + \frac{1}{2}t$ and the number of hats is given by $H(t) = 6t^{1/2}$, then how much does utility change over time? (Does this question make you go "UGH"?)

11) (16 points) For EITHER the equation in Part A OR the equation in Part B, find all stationary points. Determine if they are local maxima, local minima, or inflection points. Explain your logic.
A) $F(X) = X^3 - 6X^2 + 12X + 8$
B) $F(X) = X^3 - 12X^2 + 45X + 3$

12) (20 points) Answer EITHER Part A OR Part B.
A) Draw a Venn Diagram for Bethany College students who are graduating this week as the universal set. Draw areas for students who took four years to graduate (F) and for students who played sports (S). Given your diagram, what are the sizes of $F \cap \bar{S}$ and $F \cup S$? Explain how you reached those conclusions.
B) Suppose the sets $A = \{-1, 1, 3, 5, 7, 9, 11\}$ and $B = \{1, 2, 3, 4, 5\}$. Find A - B and $A \cup \mathbf{Z}_+$. State how you got both answers. Draw the point (4, 2, 3) on the three-dimensional graph. State how you plotted it.

13) (24 points) Answer EITHER Part A OR Part B.
A) Suppose the inverse demand function is given by $P = 200 - 2Q$ and the cost function is given by $TC = 3 + 2Q + Q^2$. Suppose the firm's plant has a maximum capacity of 30 items. Find the profit-maximizing output. If they could increase their output by 2 units, then approximately how much would their profits go up? State how you found that last answer.
B) Suppose you had the equations $X + 3Y + 4Z = 90$, $X + Y + Z = 30$, and $2X + 2Y + 3Z = 80$. Solve the system using the substitution method.

14) (26 points) Suppose your utility function is given by $U(X, Y) = XY$, find **H** and use it to determine if your utility function is strictly convex, convex, strictly concave, concave, or none of the above. Find $\bar{\mathbf{H}}$. Use it to determine if the function is strictly quasi-convex, quasi-convex, strictly quasi-concave, quasi-concave, or none of the above.

15) (28 points) Answer EITHER Part A OR Part B.
A) Suppose that 70% of the people who live in Bethany stay in Bethany (underclassmen). 20% of Bethany residents move to Pittsburgh (who knows why?) and 10% move to Wheeling. 60% of Pittsburgh residents stay in Pittsburgh while 20% move to Wheeling and 20% move to Bethany. 50% of Wheeling residents stay in Wheeling while 30% move to Pittsburgh and 20% move to Bethany. Write the **P** matrix, briefly stating how you found it. If at the beginning of the year there are 1000 people in Bethany, 5000 in Pittsburgh, and 3000 in Wheeling, then how many people will be in each location at the end of the year? How many will live in each city in two years.
B) Suppose that production of \$1 worth of energy (E) uses 30¢ of energy and 20¢ of food (F). Production of \$1 worth of food uses 10¢ of energy and 20¢ of food. Setup the Leontief Input-Output Matrix briefly stating how you found it. Use that matrix to find another matrix which will help you to find out how much of each you need to produce to sell \$110 worth of energy and \$200 worth of food. Solve the system using any matrix method. Write your answer in vector form and use it to find out how much labor you need if each dollar's worth of energy uses 2 units of labor and each dollar's worth of food uses 3 units of labor.