Wilfrid W. Csaplar Jr. Quantitative Methods... Review Sheet for Exam 1

This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

The review session will be at a time to be determined, probably Thursday, 2/14 in the normal room.

Chapter 1: What are **exogenous variables, endogenous variables, parameters, relationships, equilibrium, statics,** and **dynamics**?

Chapter 2.1: Understand was **sets, subsets,** and **elements** are. Understand set notation and the symbols $\in$**,** $\subset$**,** $\supset$**,** $\subseteq$**,** $\supseteq$**,** $\not\subset$**,** $\not\supset$**,** $\not\subseteq$**,** $\not\supseteq$, and $\notin$. Know what $\Leftrightarrow$ means. Be able to draw and interpret **Venn diagrams**. What is the symbol for the **empty set**, a.k.a. **null set**? Know how to find the sets for $\cap$ and $\cup$. What are **disjoint sets**? What is the symbol for the **complement set**? Be able to find the **relative difference of two sets**. Be able to recognize a **partition of a set**.

Chapter 2.2: Now what the following sets are: **Z, Z_+, Q,** $\Re$**,** $\Re_+$, and $\Re_{++}$. Know the following properties of real numbers: **closure, commutative laws for + and *, associate laws for + and *, distributive law, the properties of zero and one, negation, reciprocals, completeness, transitivity, reflexivity,** and **equality.** Knowing the **dimensions** (**units**) of economic variables will help you to write equations which make economic sense.

Chapter 2.3: Know how to plot an **ordered pair** on the **coordinate system (Cartesian Plain).** Know how to do a **Cartesian product** like $\{1, 2, 3\}\otimes\{4, 5, 6, 7\}$. Be able to intervals on a number line like, $[1, 4)$, $(-\infty, 7]$, etc. Know the difference between an **open interval, closed interval**, and **half-open interval**. What do **bounded** and **compact** mean? Know how to tell if a set is **convex**. (A line connecting two points is entirely in the set.) Memorize the **Euclidian distance formula** (which is easier than it looks.) Know what an **ε-neighborhood** is. Note that the definitions of a **boundary point**, **interior point**, and **convex set** easier than they look.

Chapter 2.4: Know what **function, domain, range, image,** and **value** are. Be able to tell if the function is **one-to-one, onto,** and/or **one-to-one correspondence.** (The former is the vertical line test, while the middle one is the horizontal line test, and the latter is both tests.) When is a function **invertable**? Know what a **composite mapping** is. Know what is meant by **slope coefficient** and **intercept term** in a **linear function**. An **implicit function** is basically $f(x, y)=0$. (y is a function of x, but it is not explicitly written out.) Know that **quadratic** functions have a maximum or minimum at $x = -b/2a$ where $y = ax^2 + bx + c$. **Rectangular hyperbolas** are of the form $xy=\alpha$.. Know how to plot **power functions** ($y = ax^b$) and **exponential functions** ($y=a*b^x$). Know what **logarithmic functions** are including the **natural log**. Note that for all bases (including e so it applies to the natural log ln) that $\log_b(x*y)= \log_b(x)+\log_b(y)$ which means $\log_b(x^a) = a*\log_b(x)$ and $\log_b(x/y) = \log_b(x)-\log_b(y)$. The definitions of **concave** and **convex** are fairly simple (concave looks like a cave so all points on the **secant** are below the line.). Putting **strictly** in front of them means the equality does not hold. For **quasi-concave**, the **better set** of the isobar is convex. For **quasi-convex**, the **worse set** of the isobar is convex. Note that the **Cobb-Douglas** function $f(x,y)=x^ay^b$ normally uses the assumption that $a+b<1$.

Chapter 3.1-3.2: A **sequence** is a succession of numbers of the form f(1), f(2), f(3), . . . like $f(n) = n^2$ where $n \in Z_+$ or $a_1, a_2, a_3, \ldots$ Know how to tell if it has a **limit**. (For a $n > N$, $|a_n - L| < \varepsilon$ for an arbitrarily small ε.) If there is no limit, it is **divergent**. **Definitely divergent** if the limit is ∞ or $-\infty$. It is **bounded** if there is a range the values do not exceed for large n. ($|a_n|<K$).

Chapter 3.3: Understand why $PV=FV_t/(1+r/n)^{nt}$ for **discrete compounding** and $FV_t = PV*e^{rt}$ or $PV = FV_te^{-rt}$ for **continuous compounding**. There will be more of this in Chapter 3.5.

Chapter 3.4: The limits of sequences have nine properties which are very intuitive on Pages 79 and 81. **Monotonic-**

ally increasing and **monotonically decreasing** are exactly what you would expect. It is **bounded** if there is both an upper bound and a lower bound.

Chapter 3.5: A **series** is a special type of sequence which is a summation. The standard notation is s_n where a sequence is a_n. If the $s_n = \sum a_t$ and $\lim_{n\to\infty}|a_{n+a}/a_n|=L$, then if $L< 1$ the series converges, if $L > 1$, the series diverges, and if $L = 1$, either could occur. (Check out $a_n = 0$, $a_n = 1$, and $a_n = (-1)^n$. What are s_n and $|a_{n+1}/a_n|$?) The **geometric series** is $s_n = \sum a\rho^t$ converges if $|\rho|<1$ and $\lim_{n\to\infty}s_n = a/(1-\rho)$. This can be used to prove that PV = FV $_\infty$/r and PV of an payment yearly for n years is $PV = FV_t\left(\frac{1-\left(\frac{1}{1+r}\right)^n}{1-\frac{1}{1+r}}\right)$.

The internal rate of return is the value of r which makes the equation equal.

Chapter 5.1 - 5.2: Know what **marginal analysis** means. What is a **tangent line** and how does that relate to the slope of the line? Note that the limit of the slope of the **secant line** as Δx approaches zero is the slope of the tangent line. Know the definitions of **derivative** and **total differential**. Note that marginal cost is the derivative of total cost function.

5.3: When is a function **differentiable** over [a, b]? *If the function and the plot of the slope are continuous over (a, b).*

Chapter 5.4: How much of Chapter 5.4 is on the exam will depend upon what you want. We probably won't finish this chapter, but the beginning is easy. So, you will decide on 2/18 at what point we stop. I am guessing that we will stop before the chain rule (Rule #9). **Rules for differentiation**. Remember these rules f(X) = c, then f'(X) = 0. If f(X) = mX then f'(X)=m. If $f(X) = X^n$ then $f'(X) = nX^{n-1}$. If g(X) = c*f(X) then g'(X) = c*f'(X). If h(X) = g(X) + f(X) then h'(X) = g'(X) +f'(X) and applies to adding even more terms. If h(X) = g(X)*f(X) then h'(X) = f(X)*g(X) + g'(X)*f(X) which can also be used with division by defining k(X) = 1/f(X) or you can use the division rule. If h(X) = f(X)/g(X) then h'(X) = [f'(X)*g(X) – g'(X)*f(X)]/[g(X)]2. Rule #9, if you have y = f(U) and U = g(X) the h(X) = f(g(X)) and h'(X)=f'(U)*g'(X). If the inverse of Y = f(X) is X = g(Y), then g'(Y)=1/f'(X). If $f(X) = e^x$ then $f'(X) = e^x$. If f(X) = ln(X) then f'(X) = 1/X.

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1) (25 points) Suppose the total revenue function is given by $TR = 20Q - Q^2$. Plot it over $Q \in [0, 20]$. Use the secant method to approximate the slope at Q = 8.

2) (5 points) The figure on Page 142 has an error in it. What is it? Explain your logic.

3) (10 points each) Given the information provided, find the function asked for.
A) TR = 10Q, MR =?
B) TR = P(Q)*Q where P(Q) = 20-2Q. Find MR using the product method.
C) Multiply the price function and Q (from Part B) out and find MR of that TR function.
D) $TC(Q) = 3Q^2 - 2Q +10$. ATC = TC(Q)/Q. Find the slope of ATC using the quotient rule.
E) Using the information in Part D, do the division to calculate ATC before finding its slope.
F) $TPL = L^a$. For what values of a is there diminishing MPL?

4) (10 points) If the demand curve is given by the inverse demand function P = a - bQ, then find the MR function and prove that it starts at the same place as the demand curve, but has twice the slope.