Wilfrid W. Csaplar Jr. Quantitative Methods... Review Sheet for Exam 2

This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

The review session's time will be determined, probably Wednesday. 3/26 in the normal room.

Chapter 5.4: **Rules for differentiation**. Although you will need to know the derivatives prior to the Quotient Rule, this test official starts with the **quotient rule**. If h(X) = f(X)/g(X) then h'(X) = [f'(X)*g(X) – g'(X)*f(X)]/[g(X)]2. If you have y = f(U) and U = g(X) then h(X) = f(g(X)) and using the **chain rule** h'(X)=f'(U)*g'(X). If the inverse of Y = f(X) is X = g(Y), then using the **inverse rule**, g'(Y)=1/f'(X). If f(X) = e^x then f'(X) = e^x. If f(X) = ln(X) then f'(X) = 1/X.

Chapter 5.5: Know the symbols for **second derivative** or third etc. A function is **convex** if f''(X) $\geq$ 0 and it is **strictly convex** if f''(X) >0 at all points or all but one point. Reverse the inequalities for **concave** and **strictly concave**. Note that the second derivative test tells us if we found a maximum or a minimum.

Section 6.1: What are meant by **extreme values, unconstrained, constrained, global maximum, local maximum, global minimum, local** and **minimum**? The difference between global and local maximum and minimum are global is for all x, while local is for $\hat{x} - \varepsilon \leq x \leq \hat{x} + \varepsilon$. **First order condition** is f'(x*) =0 is a necessary condition for a local maximum, local minimum, and an **inflection point** (because it gives a **stationary point**) but not a sufficient condition. Note that most of this section is just analyzing different examples. Understand what f'(x) $\neq$ 0, $\forall$ x means. (No maximum and no minimum.)

Section 6.2: **Second-order conditions** can determine if it is a maximum or minimum. f''(x*)<0 is maximum and f''(x*) > 0 is a minimum. f''(x*) = 0 could be either of those or an inflection point. Do not worry about the Taylor Series. Like Section 6.1, much of this chapter is just examples.

Section 6.3: Optimization **over an interval** is just like the local optimization except you have two differences. *First, check to make sure any extreme points you find are actually in the interval.* For example, finding the minimum of f(x) = $x^2 + 2x$, for x $\in$ [0, 100] will give a minimum at x = -1, which is not in the interval. *Second, check the limits of the interval.* In other words, check f(0) and f(100) in this example. If the maximum or minimum is inside the interval, then it is an **interior solution** and if it is not, then it is a **corner solution**. If there is a **constraint** that says x $\leq$ L or x $\geq$ L, then when L is a **binding constraint**, you get a corner solution. The **shadow price of the constraint L** is the value of f'(L).

Section 7.1: Solving **systems of linear equations** by **graphing**. Note that parallel lines have no solution, the same line twice has an infinite number of solutions. Solve by **elimination**. That is adding equations in such a method as to eliminate a variable. The second method is **substitution.** In other words, solve one equation for y or x and substitute it into the other equation. **Structural equations** are ones that are always true like $Q_D = Q_s$. Study some of the examples in the book. Know what an **underdetermined system** is.

Section 7.2: Be able to use **row operations** to solve systems of linear equations. *First, write the equations so they = a number and the variables are in the same order in all equations. Then you can multiply an equation by a number ≠ 0, add a multiple of one row to another, or switch rows.* Know how to tell if the rows are **linearly dependent or linearly independent**. If there are rows which are linearly dependent but with different values, then they are **inconsistent**. (Basically that is asking where parallel lines cross.) If the number of linearly independent equations equals the number of variables, that is good. If there are more linearly independent rows than variables, then the system is **overdetermined**. If there are fewer linearly independent rows than variables, then the system is **underdetermined**. Be able to write the equations in **matrix form**. *Basically that is a bracket then the numbers without x, y, etc., and then an end bracket. Note that if a variable is missing, you need to put in a 0.* Be able to use the same processes to get it is **reduced row-echelon form**. *That is 1 down the diagonal and 0 everywhere else except the last column.* A system of equations is **homogeneous** if all the constants after the equal sign start out as zero. Either there are an infinite number of solutions or the only solution is the **trivial solution**, all zeros. The rest of the chapter is just examples.

Non-graded Assignment #6A to be reviewed with Assignment #6.
Show all work on all questions.

1) (20 points) Suppose the demand function is given by $Q_D = 11 - P_C$ and the supply function is given by $Q_S = -4 + \frac{1}{2}P_S$. The tax rate is represented by t. What is the equation relating the two prices? What is the equation which says we are in equilibrium? Find the equation which determines the quantity produced at equilibrium as a function of t. What is the function for total tax revenue? Find the tax rate which maximizes tax revenue. Prove it is a maximum not a minimum. For each step, show all work and briefly explain what you did.

2) (20 points each) For each of the following system of equations, write it matrix form. Do row operations to get it in reduced row-echelon form to solve the system.

A) $3X + 4Y - Z = 24$,	$2X + Y - 4Z = -20$,	$X - Y - Z = 0$
B) $2X + 2Y - Z = 21$,	$2X + 4Y - Z = 10$,	$2X + 6Y - Z = 0$
C) $5X + Y - 2Z = 10$,	$X + 2Y + 4Z = 3$,	$8X + 7Y + 10Z = 19$
D) $4X - 3Y + 2Z = 11$,	$4X - 3Y + 7Z = 21$,	$2X - 3Y + Z = 1$