This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

The review session will be at a time to be determined, probably Wednesday, 4/30.

Section 8.1: What is a **matrix**? Note, we will not be dealing with game matrices. Be able to create a **Leontief Input-Output Matrix**. Remember the columns must total to less than 1 because they are the dollar amount of inputs used to produce $1 worth of goods. Know what the following terms mean: **column matrix, column vector, row matrix, row vector, square matrix, diagonal matrix, identity matrix,** and **null matrix.** (That last one is stupid.) Matrices are **equal** if every element is the same.

Section 8.2: Know how to **add** and **subtract** matrices, do **scalar multiplication** and **matrix multiplication**. Note that if A is mxn then in order to multiply AB, then B must be nxk and the product AB will be mxk. (It is possible for m to be the same as n and/or k.) *When multiplying matrices, go across the rows of the first matrix and down the columns of the second and add the products. The answer goes in the row you went across and in the column you went down.* (I suspect at this point you have done it so much, you won't get confused.) Remember, that **premultiplying** will not normally give you the same answer as **postmultiplying** partly because it is possible only one of them can be done. (*Note, that IA=AI and* $AA^{-1}=A^{-1}A$ *but you will learn the latter in Chapter 9.*) The **migration model** is just $\mathbf{x}_i=P^i\mathbf{x}_0$. (I changed the superscripts to subscripts so they do not get confused with the exponent on the P.)

Section 8.3: The **transpose** of a matrix is A^T is just rows becoming columns and vice versa. A **symmetric matrix** has $A^T=A$. For all matrices, $(A^T)^T = A$ and $(A+B)^T=A^T+B^T$. If both products are defined then $(AB)^T = B^TA^T$.

Section 8.4: An **idempotent matrix** is one where $A = A^2$. A **partitioned matrix** is one with partition(s) in it. *It works like a regular matrix for addition etc., providing that the dimensions are the same and the dimensions of the partitions are also the same.* The **trace of a matrix** is the sum of the diagonal elements. The *trace(AB) = trace(BA) providing both AB and BA are the defined, even if AB is a different dimension from BA.*

Section 9.1: An **inverse matrix** is one where $A^{-1}A=I=AA^{-1}$. If A^{-1} does not exist, then A is called **singular** otherwise it is **non-singular**. The **determinant of a matrix |A|** for a 2x2 matrix is just $a_{11}\, a_{22} - a_{12}\, a_{21}$. *If A is 2x2,* A^{-1} *is gotten by taking A, swapping the main diagonal matrix and changing the signs off-diagonal elements and then dividing by the determinant.* (Note the description on Pages 305-306 is actually the minor, cofactor, adjoint method from Section 9.3.) Note that $|A| = |A^T|$, if B is A with two rows swapped or two columns swapped, then $|A| = -|B|$ even for larger matrices. Theorems 9.3 - 9.4 can be summarized, "*if the rows or columns are linearly dependant, then the determinant is zero.*" If you add a multiple of one row (column) to another, then the determinant does not change values. The determinant of a **triangular matrix,** both **upper triangular** and **lower triangular,** is the product of the diagonal. If you multiply a row by a scalar λ, then you have multiplied the determinant by λ too. If A and B are square and the same dimensions, then $|AB|=|A||B|$. If you are solving the equation A**x**=**b**, then **x**=A^{-1}**b**. Do not worry about the geometric interpretation of the determinant, although it is twice the area between the two vectors.

Sections 9.2 and 9.3: The **minor** M_{ij} is the determinant of A after you get rid of row i and column j. The **cofactor** $C_{ij} = (-1)^{i+j} M_{ij}$. You can do the **cofactor expansion** by doing either of these:

$$|A| = \sum_{i=1}^{n} a_{ij} C_{ij} \qquad |A| = \sum_{j=1}^{n} a_{ij} C_{ij}$$

In other words, you can choose any row or any column and or expand by it. That means multiplying every element in the row or column by its corresponding cofactor. The **matrix of minors** is just a matrix where each entry is the minors of A. The **adjoint** of the matrix A is just the transpose of the **matrix of the cofactors**. Therefore, we get $A^{-1} = \text{adj}(A)/|A|$. Other important facts are $|A| = 1/|A^{-1}|$, $(AB)^{-1} = B^{-1}A^{-1}$ providing A^{-1} and B^{-1} exist. $(A^{-1})^{-1} = A$. If A is a diagonal matrix, then every entry in A^{-1} equals $1/a_{ii}$ where a_{ii} are the corresponding entries in the original matrix A.

Section 9.4: **Cramer's Rule** is simple. If you have a matrix equation A**x**=**b**, then $x_i = |A_i|/|A|$ where A_i is A with column i replaced by the vector b. Note that **Open Leontief Input-Output Matrix** is one case where Cramer's Rule can be used (but ironically the book does not) providing that you realize the equation is really (I-A)**x**=**b**. *Note, that unless there are strange values for A (like a column of A adding to more than 1), then $(I-A)^{-1}$ will only have positive values.* The **Closed Leontief Input Output Matrix** *is done just like the open one except then you multiply the transposes of the two input vectors, e^T (employees) and k^T (capital) by your final **x** to find out how much labor and capital you need to produce what is desired. Then check to see if you have enough.* Do not worry about what to do if you do not have enough labor or capital. To do that well you need to maximize output subject to constraints and that is taught in ECON 477.

Section 11.1: **Partial derivatives** are basically the same as regular derivatives except there are more than one independent variable. So, for example, $w=f(x,y,z)$. You treat the other variables as constants. *To signify partial derivative, you use ∂ instead of d.* If you want to take the derivative using the ' notation, you must put a subscript. For example $\partial f/\partial x \equiv f_1' \equiv f_x'$. *The interpretation of the partial derivative is that is the slope in the one direction. (See the diagrams on page 395.)* An **additively separate** function is one where all terms have only one of the independent variables. Note that the marginal product function (and really any marginal function) is the derivative with respect to that input. For example, MPL = $\partial TP/\partial L$ and MPK = $\partial TP/\partial K$. A **Cobb-Douglas** function is of the form $y = a * x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}$. The **constant elasticity of substitution (CES)** is of the form $y = a\left[\delta x_1^{-r} + (1-\delta)x_2^{-r}\right]^{-1/r}$. If f is a function of x and y which in turn are functions of t, $f(x(t),y(t))$, then $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$. Basically that is the chain rule from earlier in the semester (the derivative of $f(g(x))$ but written slightly different because of the partial derivatives and full derivatives.

Section 11.2: **Second-order partial derivatives** are basically the first-order partial derivatives again except that you can now have **cross-partial derivatives** which is where the first derivative is with respect to one variable and the second is with respect to a second variable. The **gradient vector ∇f** is the vector of the first partial derivatives i.e., $[f_1' \ f_2' \ldots f_n']^T$ (Note the transpose.) The **Hessian matrix** is signified by **H** or ∇^2 or as your book does it ∇_2 is the matrix of all n^2 second-order partial derivatives where element a_{ij} is f_{ij}''. **Young's theorem** says that $f_{ij}'' = f_{ji}''$ if all first and second derivatives are continuous (which is the case in 99.9% of economics). Note that if f(**x**) is additively separate, the **H** is diagonal.

11.3: The **first-order total derivative** is where you use the normal d and you do all derivatives. For example, if $y = f(x_1, x_2, x_3)$ then $dy = f_1'*dx_1 + f_2'*dx_2 + f_3'*dx_3$. If you have F(x, y) = 0 then dy/dx = $-F_x'/F_y'$. That is called **implicit differentiation** and uses the **implicit function theorem**. It can be extended to multiple variables. **Level curves, a.k.a., isobars** are lines where the function equals a constant. The most common ones in economics are **indifference curves** and **isoquants**. The slope of them can be found by subtracting the constant from F(x, y) = c from both sides and then using the implicit function theorem. The $\mathbf{MRTS_{LK}}$ is the negative of the slope of the isoquant = $-\Delta K/\Delta L$ = MPL/MPK. Similarly, the $\mathbf{MRS_{XY}} = -\Delta Y/\Delta X = MU_X/MU_Y$. Notice that in both cases the equations look upside down. Note that a **positive monotonic transformation T** of a function F will yield the same isobar map. The transformation must have T(F)>0 and T'>0. We can use that to show all Cobb-Douglas functions which have the same ratio of a to b, will have the same isobar map, thus will result in the same optimal point. Let $\tilde{u} = (1/A^k)*U^k$ where k = 1/(a+b) and $U(x_1, x_2)=Ax_1^a x_2^b$.

11.4 up through Page 441: The **second-order total differentiation** is $d^2y = f_{11}''dx_1^2 + 2f_{12}''dx_1dx_2 + f_{22}''dx_2^2$. Hint, you are just taking every derivative twice and multiplying by d whatever you took the derivative with respect to. The 2 in front of the second term is because $f_{12}'' = f_{21}''$. If $d^2y > 0$ when at least one of dx_1 or dx_2 is not zero, then it is strictly convex. *The easiest way to think about this is that in two dimensions $y=x^2$ is convex and $y''>0$.* Similarly If $d^2y < 0$ when at least one of dx_1 or dx_2 is not zero, then it is strictly concave. *If you change the > and < in this paragraph to $\geq$ and $\leq$ respectively, you eliminate the word strictly.*

Non-graded Assignment #10A to be reviewed with Assignment #10.

1) (20 points each) Find the second total differentiation of the following functions. Determine if they are strictly convex, convex, strictly concave, concave, or indeterminate. Show all work and state how you reached your conclusion.
A) $F(X, Y) = X^2 - 2XY + Y^2$
B) $F(X, Y, Z) = 10 - X^2 - Y^2 - Z^2$

2) (20 points) Suppose a utility function of xylophones (X) and yams (Y) is given by U(X, Y) = $12X^{1/2}Y^{1/3}$. Remembering that an indifference curve is given by the equation U(X, Y) = c, set this up so that you can use the implicit function theorem. Find the slope of the indifference curve at a point assuming that xylophones are on the horizontal axis. (Where else would they be?) Show all work and state how you did it.

3) (20 points) Suppose a production function is given by $Q(L, K, H) = L^{1/3}K^{/14}H^{1/4}$ where H is human capital. Use the implicit function to find $\partial L/\partial H$ along an isoquant, assuming that K is constant. What does that mean? Do not take a monotonic transformation of the function.

4) (20 points) In Question #3, what monotonic transformation could you have done if you were trying to find the slope of the isoquant? Prove that your transformation is a legitimate one. Then find $\partial L/\partial H$ along an isoquant, assuming that K is constant. Is it the same answer?