Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 240 points (to be scaled down to 200 points) and is scheduled to take 120 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give extra time but not much.

**Show all work on all questions.**

1) (6 points) For EITHER marginal utility OR marginal productivity of capital, determine the dimension (a.k.a. units) of that variable. Briefly state how you reached your conclusion.

2) (8 points) Answer EITHER Part A OR Part B.
A) The ATC(Q) function is defined as TC(Q)/Q. Find the slope of the ATC(Q).
B) TR(Q) = P(Q)*Q. Find MR.

3) (8 points) Answer EITHER Part A OR Part B.
A) Suppose your write a system of equations in the $\mathbf{Ax}=\mathbf{b}$ and in the process of finding $\mathbf{A}^{-1}$, you discovered that $|\mathbf{A}| = 0$. What does that tell you? Explain your logic.
B) What is $(((\mathbf{A}^{-1})^{T})^{-1})^{T}$? Explain your logic.

4) (10 points each) Use the matrices below to do TWO of the following calculations.
A) $3B - A^T$
B) $A^TB$
C) $AB - I$

$$A = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & -3 \\ 5 & 6 \end{bmatrix}$$

5) (12 points) Answer EITHER Part A OR Part B.
A) Plot the point (-2, 4, -1). Put dashed lines so I can see how you got to that point.
B) Plot on a number line, $5 < X \leq 10$ and $(-\infty, 4]$.

6) (12 points) Answer EITHER Part A OR Part B.
A) If U(T, W) = T*W, find the function for the slope of the indifference curve.
B) If you wanted to do a transformation on $U(M, P) = 6M^{/12}P^{1/3}$, what transformation would you make? Prove it is a valid transformation and find T(U(M, P)).

7) (12 points) Answer EITHER Part A OR Part B.
A) The curve for average fixed costs is a rectangular hyperbola. Write the formula and draw the graph.
B) The plot of real GDP over time approximates the curve $Y = Y_0e^{.025t}$. Plot that and tell me if the line is invertible. How do you know whether or not it is invertible?

8) (16 points) Answer EITHER Part OR Part B.
A) In *Principles of Macroeconomics*, we had a consumption function of C = 100 + MPC*(Y-T). Assume that taxes are 0. Let $a_n$ be the additional spending done by the $n^{th}$ person. So, for example, if somebody earned \$100, $a_1$ would be 100*MPC, Then, whoever earned that money would be $a_2 = a_1$*MPC. Therefore, $s_n$ is the geometric series which is the total additional spending by the first n people. Use this information, and the information we learned about the geometric series to find $\lim_{n\to\infty} s_n$. Hint: I am asking you to prove the formula we developed for the government spending multiplier.

B) Suppose you bought an infinitely lived bond with a face value of \$1000, a coupon rate of 8%, and interest payments made quarterly. Write the equation to calculate the present value as a sequence of payments, $a_i$. The value of the bond is the $\lim_{n\to\infty} s_i$ where $s_i$ is the $i^{th}$ entry of the geometric series. Tell me how much $\rho$ is and how you know that. Use the properties of infinitely lived geometric series to find the price of the bond.

9) (18 points) For EITHER the function in Part A OR the function in Part B, find all stationary points. Determine if it is a local maximum, local minimum, or an inflection point. Explain how you reached your conclusion.
A) $F(X) = 2X^3 - 51X^2 + 420X - 7$
B) $F(X) = X^3 - 15X^2 + 75X + 3$

10) (20 points each) Solve THREE of the following system of equations. **Solve each one with a different method.** There are four methods which I will allow for this question. They are 1) use a matrix **Ax=b** and finding $\mathbf{A^{-1}}$, 2) use elimination and substitution, 3) use Cramer's Rule, and 4) use row operations on either the equations or on the partitioned matrix. (Hints: You can always multiply an equation by a number without changing it. I would not use $\mathbf{A^{-1}}$ or Cramer's Rule on Parts C or D, the determinants are not simple for 3x3 without lots of zeros and another reason I prefer not to say.)
A) $P - Q/25 = 1$ $\quad P + Q/10 = 15$
B) $3P + 2Q = 50$ $\quad 4P - Q = 30$
C) $X + 2Y + Z = 10$ $\quad 3X - 2Y + Z = 20$ $\quad 2X + Z = 15$
D) $5X + Y - Z = 6$ $\quad X - Y + 5Z = 10$ $\quad 3X + 2Z = 10$

11) (20 points) Answer EITHER Part A OR Part B.
A) Draw a Venn Diagram with the universal set being Americans. Draw subsets V for voters and T for people who like Trump. Find on your diagram the area $\bar{V} \cap T$. Explain how you found that area and tell me the economic interpretation of that. **Given your diagram**, about what percentage of Americans do you think that is? State how you reached that conclusion.
B) Suppose $A = \{-2, 7, 9, 11\}$, $B = \{x: x \in Z_+ \text{ s.t. } x/2 \in Z\}$, and $C = \{2, 6, 7, 8, 10\}$. Find $A \cap B$, $A \cup C$, and $A \cap C$. State how you found each answer.

12) (24 points) Answer EITHER Part A OR Part B.
A) Suppose a monopoly has a demand curve of $Q = 39 - \frac{1}{2}P$ and a cost function of $TC = 2Q$. They are restricted to charging less than \$30/unit. Find their profit maximizing price. What is the shadow price of the constraint. If the price was allowed rise \$2/unit, approximately how much would the profits rise.
B) Suppose a firm has an inverse demand of $P = 20 - (1/5)Q$ and a total cost function of $TC = (1/10)Q^2 + 5Q + 10$. They are constrained to produce no more than 20 items. Find the constrained profit maximizing output. What is the shadow price of the constraint? If the quota was increased by 2, approximately how much would the profits increase?

13) (24 points) Answer EITHER Part A OR Part B.
A) Suppose producing \$1 of energy uses 20¢ of energy and 10¢ of natural resources. \$1 of natural resources uses 30¢ of energy. If we wanted to sell \$154 of each, then how much would we have to make? (This is a Leontief Input-Output Matrix problem.)
B) If the production function is given by $Q(L, K) = 16L^{1/4}K^{1/2}$ then find $\nabla Q$ and HQ. Use H to determine whether the function is positive definite, positive semi-definite, negative definite, negative semi-definite, or none of the above. How do you know that? Is it strictly concave, concave, strictly convex, convex, or none of the above? How do you know that?