This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

The review session will probably be in the Tuesday, 4/18.

Section 7.1: Solving **systems of linear equations** by **graphing**. Note that parallel lines have no solution, the same line twice has an infinite number of solutions. Solve by **elimination**. That is adding equations in such a method as to eliminate a variable. The second method is **substitution.** In other words, solve one equation for y or x and substitute it into the other equation. **Structural equations** are ones that are always true like $Q_D = Q_s$. Study some of the examples in the book. Know what an **underdetermined system** is.

Section 7.2: Be able to use **row operations** to solve systems of linear equations. *First, write the equations so they = a number and the variables are in the same order in all equations. Then you can multiply an equation by a number ≠ 0, add a multiple of one row to another, or switch rows*. Know how to tell if the rows are **linearly dependent or linearly independent**. If there are rows which are linearly dependent but with different values, then they are **inconsistent**. (Basically that is asking where parallel lines cross.) If the number of linearly independent equations equals the number of variables, that is good. If there are more linearly independent rows than variables, then the system is **overdetermined**. If there are fewer linearly independent rows than variables, then the system is **underdetermined**. Be able to write the equations in **matrix form**. *Basically that is a bracket then the numbers without x, y, etc., and then an end bracket. Note that if a variable is missing, you need to put in a 0.* Be able to use the same processes to get it is **reduced row-echelon form**. *That is 1 down the diagonal and 0 everywhere else except the last column.* A system of equations is **homogeneous** if all the constants after the equal sign start out as zero. Either there are an infinite number of solutions or the only solution is the **trivial solution**, all zeros. The rest of the chapter is just examples.

Section 8.1: What is a **matrix**? Note, we will not be dealing with game matrices. Be able to create a **Leontief Input-Output Matrix**. Remember the columns must total to less than 1 because they are the dollar amount of inputs used to produce $1 worth of goods. Know what the following terms mean: **column matrix, column vector, row matrix, row vector, square matrix, diagonal matrix, identity matrix,** and **null matrix.** (That last one is stupid.) Matrices are **equal** if every element is the same.

Section 8.2: Know how to **add** and **subtract** matrices, do **scalar multiplication** and **matrix multiplication**. Note that if A is mxn then in order to multiply AB, then B must be nxk and the product AB will be mxk. (It is possible for m to be the same as n and/or k.) <u>*When multiplying matrices, go across the rows of the first matrix and down the columns of the second and add the products*</u>. *The answer goes in the row you went across and in the column you went down.* (I suspect at this point you have done it so much, you won't get confused.) Remember, that **premultiplying** will not normally give you the same answer as **postmultiplying** partly because it is possible only one of them can be done. (*Note that IA=AI and AA^{-1}*

$=A^{-1}A$ but you will learn the latter in Chapter 9.) The **migration model** is just $\mathbf{x_i}=P^i\mathbf{x_0}$. (I changed the superscripts on the x's to subscripts so they do not get confused with the exponent on the P.)

Section 8.3: The **transpose** of a matrix is A^T is just rows becoming columns and vice versa. A **symmetric matrix** has $A^T=A$. <u>For all matrices, $(A^T)^T = A$ and $(A+B)^T=A^T+B^T$. If both products are defined then $(AB)^T = B^TA^T$.</u>

Section 8.4: An **idempotent matrix** is one where $A = A^2$. A **partitioned matrix** is one with partition(s) in it. *It works like a regular matrix for addition etc., providing that the dimensions are the same and the dimensions of the partitions are also the same.* The **trace of a matrix** is the sum of the diagonal elements. The *trace(AB) = trace(BA) providing both AB and BA are the defined, <u>even if AB is a different dimension from BA</u>.*

Non-graded Assignment #6A to be reviewed with Assignment #6.

1) (10 points) Prove that $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$ is idempotent.

2A) (10 points) When we calculated $\mathbf{\Pi} = \mathbf{p}^T\mathbf{q} - \mathbf{w}^T\mathbf{z}$, why did we transpose **p** and **w**? Use the price, quantity, factor cost, and factor vectors of $p = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}, q = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}, w = \begin{bmatrix} 12 \\ 24 \end{bmatrix}, z \begin{bmatrix} 5 \\ 10 \end{bmatrix}$.

B) (5 points) What do the 2, 20, 12, and 5 represent? State how you reached your conclusion.

3) (5 points) Prove the 2x2 null matrix is both idempotent and symmetric.

4) (25 points) Find trace(AB) and trace(BA) to prove they are the same when

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 1 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 2 & 0 \end{bmatrix}$$

5) (5 points) When we first did work with matrices, we used a partitioned matrix. What was it?

6) (40 points) Determine if the following matrices can be multiplied. If yes, then multiply them. If not, then explain why they cannot be multiplied. Answer this for AB, AC, and BC when

$$A = \left[\begin{array}{cc|c} 1 & 0 & 2 \\ -1 & -3 & -1 \\ \hline 1 & 2 & 3 \end{array}\right], B = \left[\begin{array}{cc|c} 5 & 1 & 0 \\ -1 & -2 & -3 \\ \hline 4 & 0 & 1 \end{array}\right] C = \left[\begin{array}{cc|c} -5 & -3 & -2 \\ \hline -3 & 2 & 1 \\ 1 & 2 & 3 \end{array}\right]$$