Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I cannot give extra time because some students have a class after your class.

SHOW ALL WORK ON ALL PARTS

1) (6 points each) Find the first derivative of TWO of the following.
A) $F(X) = (2X^2+X)(3X+2)^2$
B) $F(X) = (2X^3+2)/(3X+2)$
C) If the demand curve is given by $Q = (100-P)^{1/2}$, then find the slope remembering what is on the axes.

2) (6 points) For EITHER the utility function in Part A OR the utility function in Part B, find the marginal utility function and determine if there is diminishing marginal utility (MU slopes down.)
A) $U(Hats) = \ln(H^2)$

B) $U(Coats) = e^{C^{1/2}}$

3) (10 points) Answer EITHER Part A OR Part B.
A) Solve the equations for the supply and demand using the substitution method. $Q_S = -4 + 6P$ and $Q_D = 53 - (1/3)P$.
B) Solve the equations for the supply and demand using the substitution method. $4 = 6P - Q_S$ and $(1/3)P + Q_D = 53$.

4) (10 points) Answer EITHER Part A OR Part B.
A) What is the economic interpretation of dTR/dQ, d^2TR/dQ^2, and d^3TR/dQ^3? (There are two correct answers for each of the first two. Only one of them will help you with the third one, but I will accept both.)
B) Explain the reason why the first derivative test is necessary to find a maximum like when we are maximizing profits. Explain why we need the second derivative test to tell whether it is a maximum or a minimum and why that test works. Note: I am looking for statements like, "The first derivative tells us whether the curve is red or blue. We need to know the color because… The second derivative tells us whether it is sloppy. We need to know if it is sloppy because…"

5) (18 points) Answer EITHER Part A OR Part B.
A) The British government briefly considered creating an infinitely lived bond. Suppose they actually did create one with a face value of £1000 and a coupon rate of 8% paid quarterly. Write the formula for the present value of each payment, i.e., the a_n for the sequence of payments. Write the formula for the corresponding sequence s_n which shows the present value of the first n payments combined. Tell me how you found out what a_1 and ρ are. Tell me how you would find the internal rate of return if you paid £1010 for the bond. Do not worry about actually finding it.
B) When we did the government spending multiplier in *Principles of Macroeconomics*, we had government spending $1000, which led the next consumer to spend 90% of that, and which led the next consumer to spend 90% of that 90%, etc. Write the formula for the sequence of spending, a_n. How much is a_1 and ρ? State how you found that. Write the formula for the series s_n which corresponds to that sequence. Use the properties of a geometric series to prove the series converges. What does it converge to?

6) (20 points) Plot EITHER the graph in Part A OR the graph in Part B. Determine if it is differentiable everywhere. Explain your logic.

$$A)Y = \begin{cases} X & X \leq 1 \\ X^2 & X > 1 \end{cases}$$

$$B)Y = \begin{cases} 2X & X \leq 3 \\ 3X - 3 & X > 3 \end{cases}$$

7) (24 points) Answer EITHER Part A OR Part B.
A) Suppose a firm has a demand of $Q = 2400 - 10P$ and a total cost function of $TC = \frac{1}{2}Q^2 + 60Q + 20$. They are constrained to produce no more than 100 items. Find the constrained profit maximizing output. What is the shadow price of the constraint? If the quota was increased by 10, approximately how much would the profits increase?

B) Suppose a monopoly has a demand curve of $Q = 100 - 2P$ and a cost function of $TC = 10Q$. They are restricted to charging less than \$20/unit. Find their profit maximizing price. What is the shadow price of the constraint. If the price was allowed rise \$2/unit, approximately how much would the profits rise.