

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I cannot give extra time because some students have a class after your class.

SHOW ALL WORK ON ALL PARTS

1) (10 points each) Answer FOUR of the following questions using the matrices below.

A) $2A - 3B$

B) $C^T A$

C) $\text{Trace}(AD)$

D) $\text{Trace}(CD^T)$

E) B^2

F) $2C - 4D$

$$A = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 10 \\ 0 & -4 \end{bmatrix}, C = \begin{bmatrix} 7 \\ 6 \end{bmatrix}, D = \begin{bmatrix} -3 \\ 8 \end{bmatrix}$$

2) (12 points) Answer EITHER Part A OR Part B.

A) Find the value for x which makes A (on the far right) an idempotent matrix.

$$A = \begin{bmatrix} 3 & 6 \\ x & -2 \end{bmatrix}$$

B) Suppose the price of chalk, markers, and erasers are \$2/box of chalk, \$4/per box of markers, and \$1/eraser. The wage rate is \$10/hour and the rental rate for capital is \$12/hour. Setup the $p^T q - w^T z$ calculation using variables for the quantities of the products, labor and capital. Briefly state how you got the matrices. Do the calculation out.

3) (14 points) Answer EITHER Part A OR Part B.

A) Suppose 70% of Bethanians stay in Bethany while the rest move to Boston. 10% of Bostonians stay and the rest move to Bethany. Write the population matrix P . Suppose 5000 people start in Boston and 200 start in Bethany. Multiply your matrix by an appropriate matrix to find how many people will be in both locations in a year.

B) Suppose \$1 worth of energy uses \$.20 worth of energy and \$.10 worth of food to feed the employees. \$1 worth of food uses \$.30 units of energy and \$.150 worth of food. Set up the Leontief Input-Output Matrix. Use that matrix and matrix multiplication to determine how much food and energy would be used in the process of making \$1000 worth of food and \$400 worth of energy.

4) (14 points) Put the equations in EITHER Part A OR in Part B into a partitioned matrix. Use row operations to get it into row-echelon form and find all solutions.

A) $X + Y = 10$ $X + Z = 10$ $2X + 2Y + 2Z = 30$

B) $X + 2Y = 0$ $-X + 2Y + Z = 0$ $Y + Z = 0$

5) (20 points) Answer EITHER Part A OR Part B. Set it up as a partitioned matrix and use row operations to get the matrix in row-echelon form. Find all solutions.

A) The demand for umbrellas is given by $D_U = 20 + 2P_H + P_{RC} - 3P_U$. The demand for raincoats is given by $D_{RC} = 31 - 6P_H - 2P_{RC} + P_U$. The demand for hats is $D_H = 39 - 4P_H - P_{RC} + P_U$. If a store has 20 umbrellas, 10 raincoats, and 30 hats, what prices would mean supply equals demand for all three goods.

B) $4X + 3Y + Z = 26$, $X - 2Y = 12$, $-2X + 15Y + Z = -46$