

Early version for students taking *comps*.

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I cannot give extra time because some students have a class after your class.

**SHOW ALL WORK ON ALL PARTS**

1) (12 points) Answer EITHER Part A OR Part B.

A) If the production function is  $Q(K, L, H)$  where  $K$  is capital,  $L$  is labor, and  $H$  is human capital, then tell me the economic interpretation of  $\partial Q/\partial K$ ,  $F''_{KK}$ , and  $F''_{KL}$ ?

B) Tell me what the following calculations are equal to for a generic square matrices  $A$  and  $B$ .  $(A^{-1})^{-1} = ?$   
 $(AB)^{-1} = ?$

2) (14 points) Answer EITHER Part A OR Part B.

A) Use the method of finding minors, cofactors, and adjoint to find the inverse of the first matrix to the right. (Implicitly, I am asking you to prove the inverse we found for the  $2 \times 2$  matrix is correct.)

$$\begin{bmatrix} W & X \\ Y & Z \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B) Find  $A^{-1}$  for the  $4 \times 4$  matrix to the far right. Hint, this is only 14 points.

3) (16 points) Answer EITHER Part A OR Part B.

A) Suppose that \$1 of energy uses 10¢ of energy and 30¢ of food. While \$1 of food uses 20¢ of energy. Set up the Leontief Input-Output matrix and use it to find how much of each you need to produce in order to sell \$252 of energy and \$84 of food using Cramer's Rule.

B) Use Cramer's Rule to solve the system of equation  $W+X+Y+3Z=0$ ,  $3W-2X=0$ ,  $Y+2Z=0$ ,  $W+3Z=0$ .

4) (28 points) Answer EITHER Part A OR Part B.

A) Suppose  $F(X, Y, Z) = 4XY^2 + 7X^2Z$ . Find  $\nabla F$  and  $H(F)$ .

B) Suppose  $F(X, Y, Z) = 8X^3Y^2Z + Z^{1/2}$ . Find  $\nabla F$  and  $\nabla^2 F$ .

5) (30 points) One of the questions on the exam was the following system of equations. The demand for umbrellas is given by  $D_U = 100 + P_H - 2P_U$ . The demand for raincoats is given by  $D_{RC} = -2P_{RC} + 3P_U$ . The demand for hats is  $D_H = 190 - 3P_H - P_{RC}$ . If a store has 100 umbrellas, 30 raincoats, and 10 hats, what prices would mean supply equals demand for all three goods. However, this time I want you to set it up in the  $\mathbf{Ax}=\mathbf{b}$  format. Solve it **EITHER** by finding  $\mathbf{A}^{-1}$  and then multiplying the matrices appropriately **OR** by using Cramer's Rule.