

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 240 points (to be scaled down to 200 points) and is scheduled to take 120 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I can give some extra time but not much

SHOW ALL WORK ON ALL PARTS

1) (8 points) Answer EITHER Part A OR Part B.

A) For the production function $Q(L, K)$, what are the economic interpretations of $\partial Q/\partial K$, $\partial^2 Q/\partial L \partial K$ and $\partial Q/\partial L$?

B) For the equation $Q(L, K) = 18L^{1/3}K^{2/3}$, prove this function has diminishing marginal productivity of labor.

2) (10 points) What are the units of TWO of the following variables

A) Total costs

B) Marginal costs

C) Inflation

3) (12 points) Use the formal test to determine the returns to scale for EITHER $Q(L, K) = 12L^{1/3}K^{1/2}$ OR $U(H, G) = H^2 + G^2 + HG$.

4) (12 points) Answer TWO of the following parts.

A) If the population is $\text{Pop} = 100e^{.02t}$, then find $d\text{Pop}/dt$ and the growth rate of GDP.

B) If $U(G) = \ln(G^2)$ find the slope of the marginal utility function.

C) Since $\text{ATC}(Q) \equiv \text{TC}(Q)/Q$, where $\text{TC}(Q)$ is a generic function. Find the slope of the ATC function.

5) (12 points) Answer EITHER Part A OR Part B.

A) Find the marginal rate of technical substitution between labor and capital (MRTS_{LK}), for the production function $Q = 24L^{1/3}K^{1/2}H^{1/4}$ for when there are 8 units of capital, 64 units of labor, and 16 units of human capital. What does that number mean?

B) Find the marginal rate of substitution between coats and hats (MRS_{CH}) for the utility function given by $U(C, H, W) = 24C^{1/4}H^{1/2}W^{1/3}$ for when there are 64 coats, 16 hats, and 8 wallets. What does that number mean?

6) (16 points) Do TWO of the following parts. Assume $U = \{0, 1, 2, 3, 4, 5, 6\}$, $X = \{2, 4, 6\}$ and $Y = \{x \in \mathbb{Z}_+ \mid x < 5\}$

A) $X \cup Y$

B) $\bar{X} \cap Y$

C) $\bar{X} \cup (X \cap Y)$

7) (16 points) Answer EITHER Part A OR Part B.

A) Suppose that \$1 of energy uses 10¢ of energy and 30¢ of food. While \$1 of food uses 20¢ of energy. Set up the Leontief Input-Output matrix and use it to find how much of each you need to produce in order to sell \$252 of energy and \$84 of food using Cramer's Rule.

B) Use Cramer's Rule to solve the system of equation $W+X+Y+3Z=0$, $3W-2X=0$, $Y+2Z=0$, $W+3Z=0$.

8) (16 points) Find all maxima, minima, and inflection points for $Y = (X^3)/3 - 5X^2 + 21X - 10$. Use second derivative to prove what you have.

9) (18 points) Use the matrices below to find THREE of the following answers.

$$A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 5 & -2 \\ 10 & -5 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, D = \begin{bmatrix} 10 \\ 20 \end{bmatrix}, E = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

- A) $2A + B$
- B) AC
- C) $E^T C$
- D) $\text{Trace}(DE^T)$ and $\text{Trace}(D^T E)$
- E) $|A| + |B|$

10) (18 points) Answer EITHER Part A OR Part B.

A) In ECON 162, we discussed the government spending multiplier. The change in GDP was equal to $\Delta G, MPC * \Delta G, MPC * MPC * \Delta G$, etc.. Find the sequence a_1, a_2 , etc. Then find the series s_1, s_2 , etc. Find $\lim_{t \rightarrow \infty} (s_t)$. State how you found the limit. Does your answer agrees with what we calculated in ECON 162?

B) Suppose a bond has a face value of \$1000, a coupon rate of 8%, pays interest quarterly, and matures in $4\frac{1}{2}$ years. Set up the sequence a_1, a_2 , etc. Then find the series s_1, s_2 , etc. Write the equation out which would be able to tell you the internal rate of return if you paid \$1010 for it. State how you decided what number went in which spot.

11) (18 points) For EITHER the equation in Part A OR the equation in Part B, find H and \bar{H} . Write out the test you would need to do to determine if the function is concave, convex, semi-concave, semi-convex or none of them. Also, write out the test to determine if it is quasi-concave, quasi-convex, or neither. Do not actually do the test. So, if the test is $|K| > 0$, then write out the K matrix, but do not take the determinant.

A) $U(J, P) = J^{1/2} + P^{1/2} + JP$

B) $U(G, T) = (GT)^2$

$$\begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 3 \\ 0 & -3 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 4 \\ 3 & 0 & 1 \end{bmatrix}$$

12) (20 points) Find A^{-1} for EITHER of the matrices to the right.

13) (20 points) Solve the following system of equations. You can use graphing method, substitution method, elimination method, or matrix row operations. (I doubt you will use the graphing method.)
 $3X + 2Y + 5Z = 7, X - 3Y + 2Z = 3, X + 8Y + Z = 1$

14) (20 points) Draw a Venn Diagram where the universal set is Bethany students. Have G be the set of students in a Greek organization and S be the set of students in sports. Given your diagram, what is the size of $G \cap S$? Explain how you found that.

15) (24 points) Suppose the total cost function is given by $TC = 10 + 100Q + 2Q^2$ and the demand curve is given by $P = 50 - Q/2$. Suppose there is a price ceiling of \$40 per unit. Find the profit maximizing price and quantity. What is the shadow price of the ceiling? What does that mean?