Wilfrid W. Csaplar Jr. Quantitative Methods... Review Sheet for Final Exam

This review sheet is intended to cover everything that could be on the exam; however, it is possible that I will have accidentally left something off. You are still responsible for everything in the chapters covered except anything that I explicitly say you are not responsible for. Therefore, if I left something off of this sheet, it can still be on the exam. There will be no multiple-choice questions. Most of the questions will be like the ones in the homework assignments, and possibly a few definition questions, but I am more likely to ask questions that make you use the definitions rather than recite them. I will probably ask one of the questions from the book at the end of the chapters.

When I write the final, I try do draw questions evenly from all parts of the course. However, Sections 11.4 - 11.5 had not be covered before, so there will be slightly more emphasis on those three sections.

The review session will probably be 5/13 or 5/14. The final is Tuesday 5/15 at 10:00

11.4: The **second-order total differentiation** is $d^2y = f_{11}''dx_1^2 + 2f_{12}''dx_1dx_2 + f_{22}''dx_2^2$. Hint, you are just taking every derivative twice and multiplying by d whatever you took the derivative with respect to. The 2 in front of the second term is because $f_{12}'' = f_{21}''$. If $d^2y > 0$ when at least one of dx_1 or dx_2 is not zero, then it is strictly convex. *The easiest way to think about this is that in two dimensions $y=x^2$ is convex and $y''>0$.* Similarly If $d^2y < 0$ when at least one of dx_1 or dx_2 is not zero, then it is strictly concave. *If you change the > and < in this paragraph to $\geq$ and $\leq$ respectively, you eliminate the word strictly.* The easiest way to do the tests for strictly concave, concave, strictly convex, and convex, is to use the theorem on Page 443, if **H** is **positive definite** then f(**x**) is strictly convex, if **H** is **negative definite** then f(**x**) is strictly concave, if **H** is **positive semi-definite** then f(**x**) is convex, and if **H** is **negative semi-definite** then f(**x**) is concave. Unfortunately, this requires a little bit of Section 10.3 which we skipped. Basically if $|H_i| > 0$ for all i, **H** is positive definite. If $|H_i| \geq 0$ for all i, **H** is positive semi-definite. If $|H_i| <0$ for odd i, and $|H_i| >0$ for even i, then **H** is negative definite. If $|H_i| \leq 0$ for odd i, and $|H_i| \geq 0$ for even i, then **H** is negative semi-definite. Here, H_i is the ixi matrix which is the i upper left-hand rows and columns of **H**. The positive definite and positive semi-definite is easy to remember. For the negative, think of $(-1)^i$. Starting with -1, it alternates from negative to positive. *If f(**x**) is additively separate, then you have a diagonal matrix and everything is much easier.* The easiest way to do the tests for strictly concave, concave, strictly convex, and convex, is to use the theorem on Page 443, if **H** is **positive definite** then f(**x**) is strictly convex, if **H** is **negative definite** then f(**x**) is strictly concave, if **H** is **positive semi-definite** then f(**x**) is convex, and if **H** is **negative semi-definite** then f(**x**) is concave. Unfortunately, this requires a little bit of Section 10.3 which we skipped. Basically if $|H_i| > 0$ for all i, **H** is positive definite. If $|H_i| \geq 0$ for all i, **H** is positive semi-definite. If $|H_i| <0$ for odd i, and $|H_i| >0$ for even i, then **H** is negative definite. If $|H_i| \leq 0$ for odd i, and $|H_i| \geq 0$ for even i, then **H** is negative semi-definite. Here, H_i is the ixi matrix which is the i upper left-hand rows and columns of **H**. The positive definite and positive semi-definite is easy to remember. For the negative, think of $(-1)^i$. Starting with -1, it alternates from negative to positive. *If f(**x**) is additively separate, then you have a diagonal matrix and everything is much easier.*

Section 11.5: A **bordered Hessian matrix** is represented by $\bar{\mathbf{H}}$ is **H** with a row and column added to the upper and left sides. They are $[0\ \ f_1'\ \ f_2'\ \dots\ f_n']$. It can be useful because if $\bar{\mathbf{H}}$ is such that all $|\bar{H}_i| < 0$ for odd i and >0 for even i, then f is quasi-concave. (See Section 2.4 if you forgot that.) If $|\bar{H}_i| < 0\ \forall$ i then f is quasi-convex. Note that $\bar{H}_i$ is H_i with the border added, so $\bar{H}_1$ is 2x2 not 1x1. *Therefore, $|\bar{H}_1|$ must be negative*. Therefore, for quasi-convexity you just remember that it is the negative the whole way through. **Homogeneous of degree k** is found by replacing all x_i with cx_i. If you can then get the c out of the new function resulting in the following $f(c\mathbf{x}) = c^k f(\mathbf{x})$ then it is homogenous of degree k. *The value of k is helpful. If k<1 then there is decreasing returns to scale **(DRTS)**, if k=1 then there are constant* *returns to scale* ***(CRTS)****, and if k>1 then there are increasing returns to scale* ***(IRTS)***. Note that you cannot take a positive monotonic transformation before finding the degree because that would change the degree.

Non-graded Assignment #10B to be reviewed during the last class.

1) (10 points) Find the bordered Hessian for $f(x, y) = 4x^{1/2}y^{1/2}$.

2) (20 points) Find $\bar{\mathbf{H}}$ for $f(x, y) = 5xy$. Determine if f is quasi-concave, quasi-convex, or neither.

3) (20 points) Find the degree of homogeneity for the general Cobb-Douglas production function $Q = A*K^aL^b$. What does that tell you about a simple way to tell the returns to scale for a Cobb-Douglas? Now do the positive monotonic transformation where $\mathrm{Q} = (1/A^c)Q^c$ where $c = 1/(a+b)$. Find the degree of homogeneity for that new function.

4) (10 points) Given that taking a positive monotonic transformation of a function does not affect the level curves, and given what we already learned about Cobb-Douglas functions in class, Question #2 and Question #3, what can you say about the quasi-concavity or quasi-convexity of all Cobb-Douglas functions? Explain your logic.

5) (20 points) Find $\bar{\mathbf{H}}$ for $f(x, y) = x^2 + y^2$. Determine if f is quasi-concave, quasi-convex, or neither.

6) (20 points) Find $\bar{\mathbf{H}}$ for $f(x, y) = (x + y)^2$. Determine if f is quasi-concave, quasi-convex, or neither.