

Write your name on the cover of the test booklet and nowhere else. Failure to follow these directions will cost you 1 point. The test has 150 points (to be scaled up to 200 points) and is scheduled to take 75 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I will allow some extra time, but I will not allow much.

Show all work on all questions.

1) (8 points) Do EITHER Part A OR Part B.

A) Why do we do the "returns-to-scale" test when testing for a valid utility function?

B) Suppose we wanted to minimize your income (so you can work as little as possible) subject to your utility is $U = 100$. What is the economic interpretation of λ ?

2) (10 points) Do EITHER Part A OR Part B.

A) Is the $U(X, Y) = 8X^{1/2} + 8Y^{1/2}$ a legitimate utility function? Prove your answer is true using the formal proof.

B) Suppose the current interest rate is 5%. Your income this year is \$200, next year is \$300, and in two years will be \$400. Your utility function is $U = 16(C_0C_1C_2)^{1/4}$. Setup the Lagrangian and BRIEFLY state how you got it. **Do not solve it.**

3) (20 points) Find all Nash equilibria in the following matrix, if any exist. Prove that you found all and prove they are Nash equilibria. Does either firm have a dominant strategy? How can you tell? Find the cooperative equilibrium. Explain how you found it.

Payoff Matrix		Pirates	
		High price	Medium Price
Red Sox	High Price	7 14	8 19
	Low Price	13 15	12 18

4) (30 points) Answer EITHER Part A OR Part B.

A) Suppose your utility function is $U(H, C) = 4H^{1/2}C^{1/4}$. The price of a hat is \$20/hat and the price of a coat is \$40/coat. Find the utility maximizing consumption of hats and coats if you're your budget is \$1920. How much is your utility? Show all work. **Do not worry about λ .**

B) Suppose your utility function is $U(B, G) = 3B^{1/3}G^{1/3}$. The price of a ball is \$10/ball and the price of a glove is \$80/glove. Find the utility maximizing consumption of balls and gloves if you're your budget is \$1280. How much is your utility? Show all work. **Do not worry about λ .**

5) (40 points) Answer EITHER Part A OR Part B.

A) Suppose your utility function is given by $U(D, M, H) = (D \cdot M \cdot H)^{1/4}$. The price of a dinner (D) \$20/meal, price of a movie (M) is \$10/ticket, and a hat (H) costs \$40/hat. Your income is \$400. You also have a time constraint of a dinner takes 3 hours while a movie takes 1.5 hours. You have 24 hours of time to spend. Find the utility maximizing levels of D, M, and H. Show all work. **Do not worry about finding λ . Show all work.**

B) Suppose the Cournot firm's best response function is given by $Q_1 = 40 - \frac{1}{2}Q_2$, and the leader's profit is given by $\Pi_L = -Q_L^2 - 3Q_LQ_F + 60Q_L - 23$. Set up the Lagrangian for the Von Stackelberg leader and state how you found it. Find the equilibrium outputs for the two firms and the price sold. The industry demand is $P = 200 - 2(Q_1 + Q_2)$. **Do not worry about finding λ . Show all work.** (Because this is rigged, the difference between Q_L & Q_F is too big.)

6) (42 points) Answer EITHER Part A OR Part B.

A) Suppose that both firms are facing the following demand and total cost functions: $P = 121 - 2(Q_1 + Q_2)$ and $TC_i = 16 + Q_i + 3Q_i^2$. Use this to derive the best response function for Firm 1 and the equilibrium output for each firm, assuming that the firms are Cournot style firms. What are the outputs of the two firms and the market price? Set up the Von Stackelberg problem. Start solving it, but **stop after you take all of the derivatives. Show all work.**

B) Suppose that labor costs \$8/hour and capital costs \$27/unit and the firm's production function is given by $Q = 6K^{1/3}L^{1/3}$. Derive the total cost function which maximizes profits for a given output Q. **What is the value of λ ?** What are the marginal cost and average total cost functions? **Show all work.**