

Place your name on the back of this sheet of paper and nowhere else. Staple your answers face up on the front of this sheet of paper. Failure to follow these directions will cost you 10 points. If you use double-sided printing or print on the back of scrap paper, I will give you one additional point.

Show all work on all questions.

1) (5 points) Suppose the utility function is given by $U(X) = \ln(X)$. Find the marginal utility function and the slope of the marginal utility function.

2) (10 points each) Answer each part separately. Find the first and second derivatives of the following equations.

A) $\text{Population}(t) = 300e^{0.2t} - 100e^{0.1t}$

B) $F(X) = (3X^2 + 2X + 4)^{1/2}$

3) (15 points) Suppose the total cost function is given by $TC(Q) = 5 + 4Q + \frac{1}{4}Q^2$. The demand curve is given by $Q = 86 - \frac{1}{4}P$. Find the profit function. Find the profit-maximizing quantity. Prove it is profit-maximizing.

4) (15 points) Note that I do not give specific equations for this question, because I want you to prove it in general. Suppose the inverse demand function is given by $P(Q)$ and the total cost function is given by $TC(Q)$. What are the formulas for MR and MC? Write the equation for the profit function. Find the equation for the profit-maximizing quantity. Use it to prove $MR=MC$ is profit maximizing.

5) (15 points) Suppose the $TP(L) = Q = L^{1/2}$ and the inverse demand curve is given $P = 200 - Q$. Find the total revenue function. $MRP(L)$ is defined as how much revenue is gotten by hiring one more unit of labor. Find that function.

6) (15 points) Suppose the revenue from a tax is given by $TR = 100t - t^2$ where t is the tax rate. Find the tax rate which maximizes tax revenue. Prove it is a maximum.

7) (15 points) Suppose the demand curve is given by $Q = 300 - \frac{3}{8}P$. Find the quantity which maximizes total revenue. Prove it maximizes revenue.