Wilfrid W. Csaplar Jr., Ph.D. Economics 205 Exam #2 2020/2/28

Write your name on the cover of the test booklet and nowhere else. Enclose this sheet with the booklet. Failure to follow these directions will cost you 1 point. The test has 100 points (to be scaled up to 170 points) and is scheduled to take 50 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I cannot give extra time because some students have a class after your class.

Show all work on all questions.

1) (8 points each) Find the required derivatives for TWO of the following.
A) Suppose the utility function is given by $U(T) = \ln(3T^2)$. Find the slope of the marginal utility function.
B) ATC = TC(Q)/Q. Find the slope of the ATC.
C) If the total product of labor is given by $Q(L) = 4(9+2L^2)^{1/2}$, then find the marginal product of labor.

2) (8 points) For EITHER the sequence in Part A OR the sequence in Part B, determine if it converges or diverges. Prove your conclusion is true. If it converges, find what it converges to.

A) $a_n = \dfrac{4+2n+5n^2}{10n^2+3n+6}$ B) $a_n = \dfrac{6+2n+n^2}{4n^3+3n+7}$

3) (12 points) Answer EITHER Part A OR Part B.
A) Suppose you were to buy a bond with a face value of \$5000, a coupon rate of 4%, with interest paid semi-annually, and a maturity of 2½ years after you bought it. Set up the equation which will determine the internal rate of return if you paid \$5010 for it. State how you determined which number goes where. Do not solve it. Is that return greater, less than, or equal to 4%? Explain your logic.
B) Suppose you were to buy a bond with a face value of \$10,000, a coupon rate of 6%, with interest paid quarterly, and a maturity of 6½ years after you bought it. Set up the equation which will determine how much you should pay if you want an internal rate of return of 7%. State how you determined which number goes where. Do not solve it. Is that number going to be greater, less than, or equal to \$10,000? Explain your logic.

4) (12 points) Answer EITHER Part A OR Part B.
A) For the function $F(X) = 2X^3 – 3X^2 – 36X + 6$. Find all stationary points. Determine if they are max min or inflection points. Prove your conclusion.
B) Without plotting it, use the derivative(s) to determine if the function $Y = 4e^{(3-2X)}$ is convex, concave, strictly convex, strictly concave, or none of the above. Briefly explain how you reached your conclusion.

5) (12 points) Answer EITHER Part A OR Part B.
A) In *Principles of Macroeconomics*, we calculated the government spending multiplier. We started with the equation C = 100 + MPC*(Y-T). We assumed that T was zero. We then had a sequence of expenditures, ΔG, ΔC, ΔC ... where ΔC = Δ Y*MPC where ΔY equaled the previous entry in the sequence. Write this sequence in the form of a geometric sequence, i.e., an = ... Make sure you give the formula for at least the first four elements in the sequence and for the nth element. Then write the formula for the first two entries in geometric series sn. Given the properties of geometric series, what is the value for value of $\lim_{n\to\infty}(\sum\Delta Y) = \lim_{n\to\infty}(sn)$? Since the multiplier is $(\sum\Delta Y)/\Delta G$, what is the formula for the multiplier? If the MPC = .9, then how much is that?
B) In macroeconomics, we do not discuss this, but there is a money multiplier. In the process, the Fed will increase the reserves by buying a bond of value B. The bank loans out that amount so the money

supply increases by B = ΔM. The bank which gets that deposit will make a loan of the amount (1-rrr)*B where rrr is the required reserve ratio. The change in the money supply in that step ΔM = (1-rrr)*B. The next bank loans out (1-rrr) times that amount, i.e., $(1\text{-rrr})^2$*B. This process continues. Write this sequence in the form of a geometric sequence, i.e., an = ... Make sure you give the formula for at least the first four elements in the sequence and the nth element. Then write the formula for the first two entries in the geometric series sn. Given the properties of geometric series, what is the value for $\lim_{n\to\infty}(\sum\Delta M) = \lim_{n\to\infty}(sn)$? Since the multiplier is (∑ΔM)/B, what is the formula for the multiplier? If the rrr = .1, then how much is that?

6) (20 points) Answer EITHER Part A OR Part B.

A) Suppose the income taxes were given by the function to the right. Plot the taxes as a function of income. Is it differentiable everywhere? Explain your logic.

$$\begin{cases} I < 100 & .2I \\ 100 \le I < 200 & 20 + .3(I - 100) \\ I \ge 200 & 50 + .4(I - 200) \end{cases}$$

B) In upper-level Microeconomic courses, you will find out about total factor costs (TFC) and marginal factor costs (MFC). Suppose that the $TFC(L) = 5(2+4L)^3$. Find the MFC and the slope of the MFC.

7) (20 points) Answer EITHER Part A OR Part B.

A) Suppose a monopoly has a demand curve of Q = 31 - ¼P and a cost function of TC = 12Q. They are restricted to charging less than $60/unit. Find their profit maximizing price. What is the shadow price of the constraint. If the price was allowed rise $2/unit, approximately how much would the profits rise.

B) Suppose a firm has an inverse demand of Q= 1200-4P and a total cost function of $TC=\frac{1}{4}Q^2+50Q+10$. They are constrained to produce no more than 200 items. Find the constrained profit maximizing output. What is the shadow price of the constraint? If the quota was increased by 10, approximately how much would the profits increase?