

Do NOT write your name anywhere. (Canvas will tell me who turned in the exam.) Take pictures of your answers and use your own software or <https://pdfcandy.com/> to create a PDF for each answer which requires an upload. If it is large, resize it to A4. Upload that to Canvas. Upload each answer as a separate file with that question. Failure to follow directions will cost you one point.

You are not allowed to use your books, notes, the internet, or other people when taking this test. You can use the internet to access Canvas and to convert your answers to PDF files. Nothing else.

If you run out of time or lose your internet connection, you can do a second submission. You do NOT have to redo the questions you already did. I will be able to see every submission. If you have problems, you can always contact me via Zoom or e-mail. If you use Zoom, open it in a new tab or window.

Failure to follow these directions will cost you 1 point. The test has 240 points (to be scaled down to 200 points) and is scheduled to take 120 minutes. Therefore, expect to spend 1 minute for every 2 points. For example, a 12-point question should take 6 minutes. I have it set up to only give you an hour and a half.

Show all work on all questions.

1) (6 points) For EITHER the unemployment rate OR the national debt, what are its units, a.k.a. its dimension? State how you reached your conclusion.

2) (8 points) Answer EITHER Part A OR Part B.

A) Find the distance between $(4, -3)$ and $(-1, 9)$.

B) Plot both $X < -3$ and $(1, 4]$

3) (8 points) Answer EITHER Part A OR Part B.

A) Prove $\begin{bmatrix} -3 & -6 \\ 2 & 4 \end{bmatrix}$ is idempotent.

B) Find the trace of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

4) (12 points) Use $A = \begin{bmatrix} 2 & 1 \\ -3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -1 \\ 0 & -4 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 6 \\ -2 & -5 \end{bmatrix}$ to find EITHER $2A - B^T$

OR $B^T - 3C$.

5) (14 points) Answer EITHER Part A OR Part B.

A) Suppose that in one year, 80% of the people from Bethany stay in Bethany and the rest move to Pittsburgh. 90% of people from Pittsburgh stay there and the rest move to Bethany. Setup the migration matrix P . Use that to find the matrix which could be used to figure out where people

will be in two years. Note, that I am not asking for how many people are in each place because that would require the initial population. I am asking for the matrix which gives percentages in two years, just like P gives for one year.

B) Suppose the prices of gloves, masks, and toilet paper are \$4/glove, \$3/mask, and \$2/sheet. The quantity of each sold are 100 gloves, 200 masks, and 50 sheets of toilet paper. Suppose workers get paid \$20/hour and capital owners get \$30/hour. We used 25 hours of labor and 10 hours of capital. Setup the $p^Tq - w^Tz$ matrix calculation for finding profits. Do out the calculation.

6) (16 points) Two common utility functions are $U(M) = \ln(M)$ and $U(T) = T^{1/2}$. Plot ONE of them. You only need to plot two exact points and get the general shape correct. Make sure you label the two points. (M is masks and T is toilet paper.)

7) (16 points) Answer EITHER Part A OR Part B.

A) Find the formula for the slope of the isoquant, a.k.a. level curve, a.k.a. isobar, for $Q(L, K) = 6L^{1/2}K^{1/3}$. L is graphed on the X-axis.

B) Find the formula for the slope of the indifference curve, a.k.a. level curve, a.k.a. isobar, for $U(M, T) = 12M^{1/4}T^{1/3}$. M is graphed on the X-axis.

8) (16 points) Answer EITHER Part A OR Part B.

A) In *Principles of Macroeconomics*, we calculated the government spending multiplier. We started with the equation $C = 100 + MPC*(Y-T)$. We assumed that T was zero. We then had a sequence of expenditures, $\Delta G, \Delta C, \Delta C \dots$ where $\Delta C = \Delta Y * MPC$ where ΔY equaled the previous entry in the sequence. Write this sequence in the form of a geometric sequence, i.e., $a_n = ar^{n-1}$. Make sure you give the formula for at least the first four elements in the sequence and for the nth element ($a_1, a_2, a_3, a_4,$ & a_n). Then write the formula for the first two entries in geometric series (s_1 and s_2). Given the properties of geometric series, what is the value for value of $\lim_{n \rightarrow \infty} (\sum \Delta Y) = \lim_{n \rightarrow \infty} (s_n)$? Since the multiplier is $(\sum \Delta Y) / \Delta G$, what is the formula for the multiplier? If the $MPC = .9$, then how much is that?

B) In Business Cycles & Forecasting this semester, but there is a money multiplier. For this exam, assume people keep no cash on hand and no excess reserves. In the process, the Fed will increase the reserves by buying a bond of value $\Delta BASE$. The bank loans out that amount so the money supply increases by $\Delta M = \Delta BASE$. The bank which gets that deposit will make a loan of the amount $(1-res)*\Delta BASE$ where res is the required reserve ratio. The change in the money supply in that step $\Delta M = (1-res)*\Delta BASE$. The next bank loans out $(1-res)$ times that amount, i.e., $(1-res)^2*\Delta BASE$. This process continues. Write this sequence in the form of a geometric sequence, i.e., $a_n = ar^{n-1}$. Make sure you give the formula for at least the first four elements in the sequence and for the nth element ($a_1, a_2, a_3, a_4,$ & a_n). Then write the formula for the first two entries in geometric series (s_1 and s_2). Given the properties of geometric series, what is the value for value of $\lim_{n \rightarrow \infty} (\sum \Delta M) = \lim_{n \rightarrow \infty} (s_n)$? Since the multiplier is $(\sum \Delta M) / \Delta BASE$, what is the formula for the multiplier? If the $res = .1$, then how much is that?

9) (16 points) Answer EITHER Part A OR Part B.

A) Find all maxima, minima, and inflection points of $Y = (X^3)/3 - 3X^2 + 8X - 5$. Use the second derivative test to determine which it is.

B) If the population at time t is given as $\text{Pop}(t) = 400e^{0.04t}$, then find the growth rate of the population, i.e., $\Delta\text{Pop}/\text{Pop}$, as a function of time.

10) (20 points) A valid utility function has two requirements. The first is diminishing marginal utility of all goods. The second is decreasing returns to scale, i.e. it is homogeneous of degree k where $0 < k < 1$. Prove that $U(M, T) = 32M^{1/4}T^{1/2}$ is a valid utility function.

11) (20 points) Answer EITHER Part A OR Part B.

A) Find \bar{H} for $f(x, y) = x^2 + y^2$. Determine if f is quasi-concave, quasi-convex, or neither.

B) Find \bar{H} for $f(x, y) = (x + y)^2$. Determine if f is quasi-concave, quasi-convex, or neither.

12) (20 points) Draw a Venn Diagram where the universal set is Bethany College students. Have subsets people from West Virginia (W), people from Pennsylvania (P), and athletes (A). Shade in EITHER the area for $(W \cap \bar{A}) \cup P$ OR $(P \cup W) \cap \bar{A}$. Briefly explain how you found it. What is the economic interpretation of that area?

13) (20 points) For EITHER Part A OR Part B, put the system of equations into a partitioned matrix and use row operations to solve the system of equations.

A) $4X - 3Y - Z = 0$, $8X + 2Y - 10Z = 0$, $8Y - 8Z = 3$

B) $X + 2Y - Z = 4$, $2X - 2Y + 4Z = 2$, $4X + 2Y + 2Z = 10$

14) (24 points) Answer EITHER Part A OR Part B.

A) Suppose total costs = $20 + 2Q + (1/4)Q^2$ and the demand curve is given by $Q = 100 - P/2$. Find the profit maximizing price and quantity if there is a quota of 40. What is the shadow price of the constraint? What does that mean?

B) Suppose the royalties for a book are 10% of the revenue. The demand curve is given by $Q = 100 - P/4$ and the total cost of printing the book is $20 + 36Q$. Find the profit maximizing quantity and price. Find the quantity and price which the author would want the book to sell for.

15) (24 points) Answer EITHER Part A OR Part B.

A) Solve the system of equations $X + Y + Z = 60$, $2Y - Z = 10$, $Z = 30$ by setting up a 3×3 matrix for $\mathbf{Ax}=\mathbf{b}$ and then using the \mathbf{A}^{-1} method. (I realize it is easier to use the substitution method, but I want you to do it this way.)

B) Suppose \$1 of energy uses \$.10 of energy and \$.20 of food to produce. \$1 of food uses \$.30 of food to produce. Setup the Leontief input-output matrix. Use it to find out how much of each you need to produce if you want to sell \$1260 of energy and \$1890 of food, using Cramer's Rule.